Beyond the Fourier Transform :
Coping with Nonlinear, Nonstationary Time Series

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Seminar Announcement, Johns Hopkins University, 1998

SEMINAR NOTICE

Mode analysis of non-stationary, random systems; the overthrow of Fourier analysis.

Dr Norden Huang,
NASA Goddard Space Flight Center

Modern techniques for the analysis of non-stationary random systems and the identification of embedded characteristic structures include wavelet analysis and Hilbert transforms, each of which has its own limitations. A new and simple method applicable to non-linear systems will be described; it involves an effectively finite and often small number of discrete modes and gives sharp identification of embedded structures. Examples will be given of synthetic records analysed by various methods, and of real time series of non-linear systems, such as surface waves, tidal records and low-frequency oceanic oscillations; this new technique gives much simpler and more revealing interpretations than conventional methods do.

12 noon, Mon. 5 February, 304 Olin, JHU

Morton K Blaustein Department of Earth & Planetary Sci.

For Further information: Owen Phillips 516-4658

Jean-Baptiste-Joseph Fourier

1807 ***"On the Propagation of Heat in Solid Bodies"***

1812 ***Grand Prize of Paris Institute***

"Théorie analytique de la chaleur"

'... the manner in which the author arrives at these equations is not exempt of difficulties and that his analysis to integrate them still leaves something to be desired on the score of generality and even rigor.'

1817 ***Elected to Académie des Sciences***

1822 ***Appointed as Secretary of Math Section***

paper published

Fourier's work is a great mathematical poem.

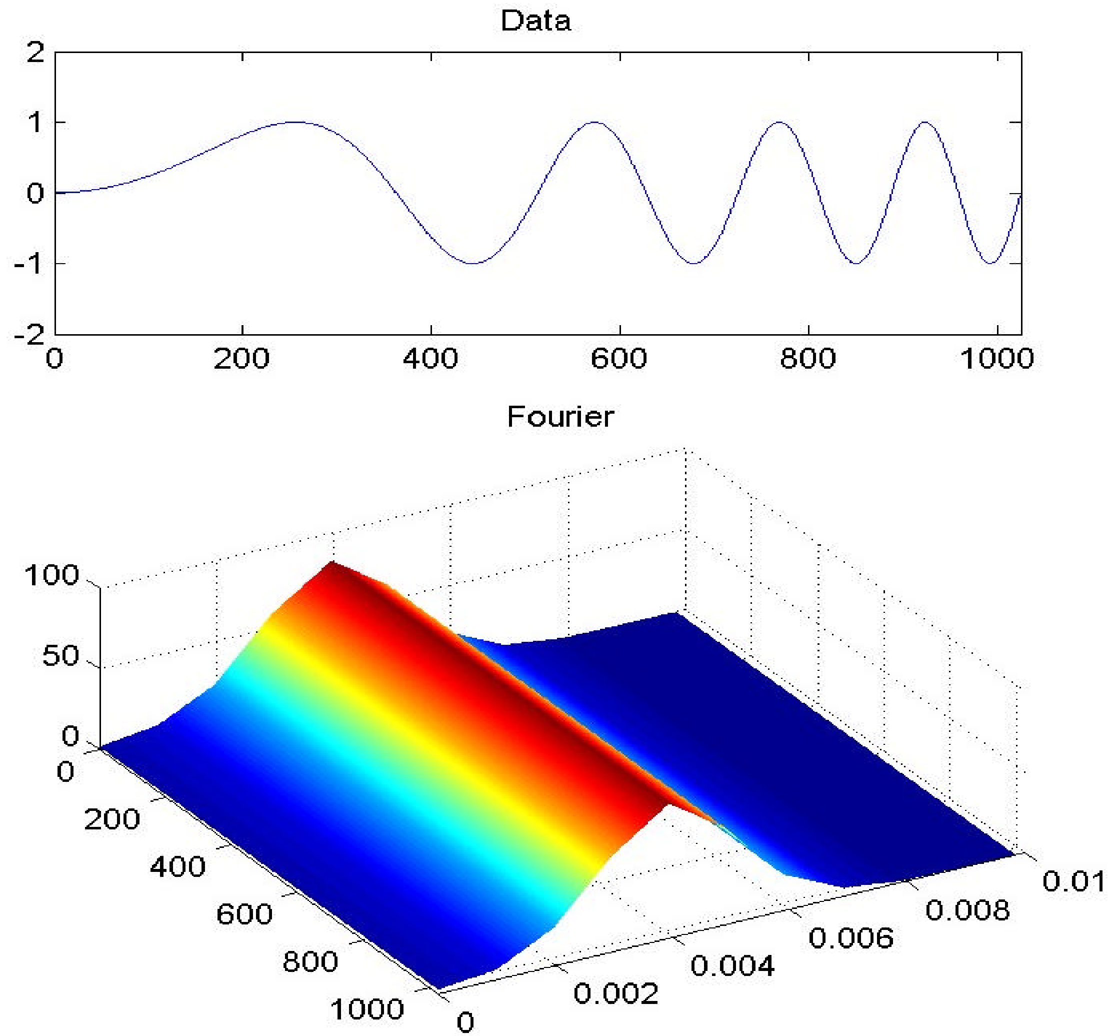
Lord Kelvin

Fourier Integral

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t}\, dt \; ;$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{-i\omega t}\, d\omega$$

Fourier Spectrum

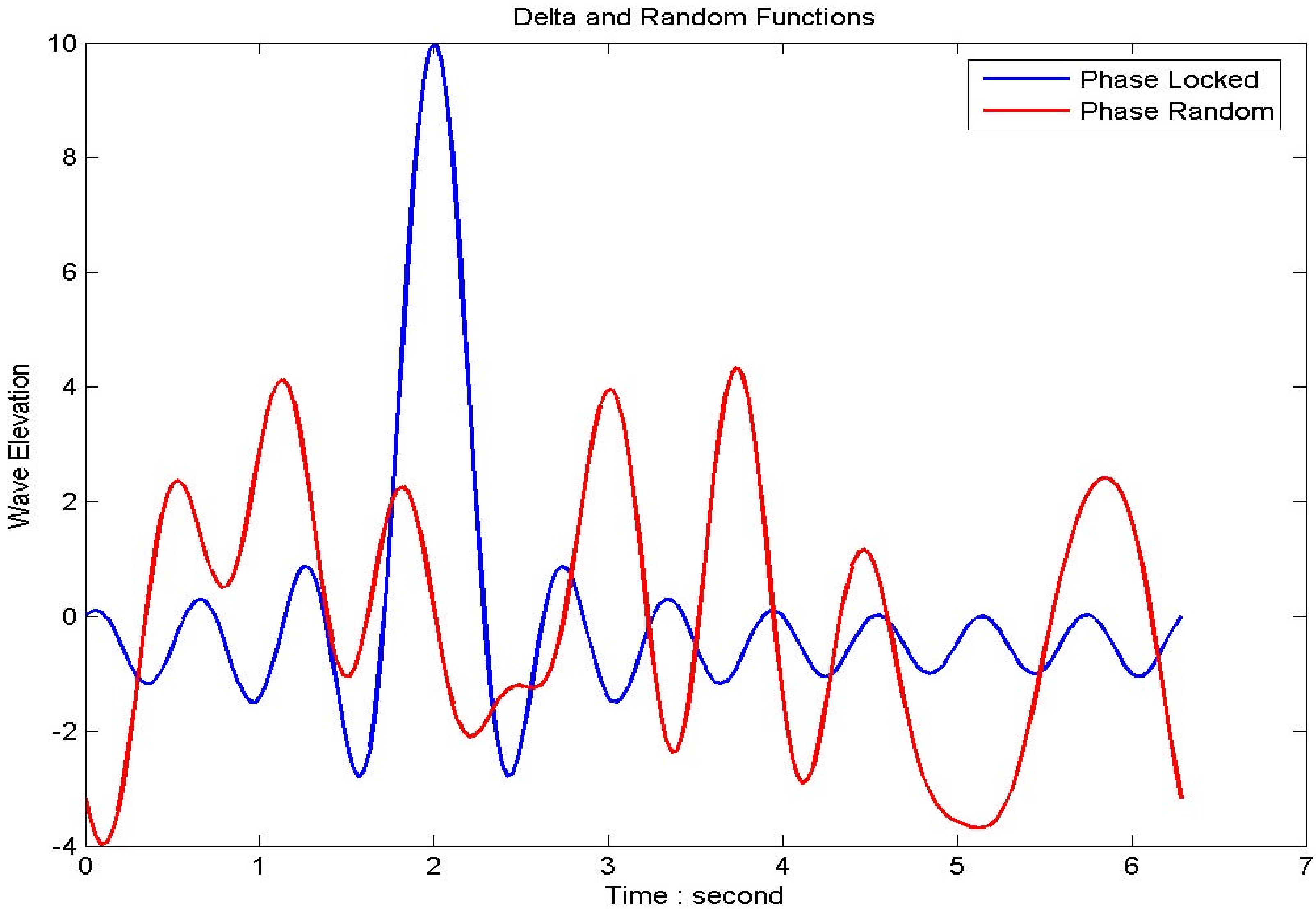


Fourier Series Expansion:

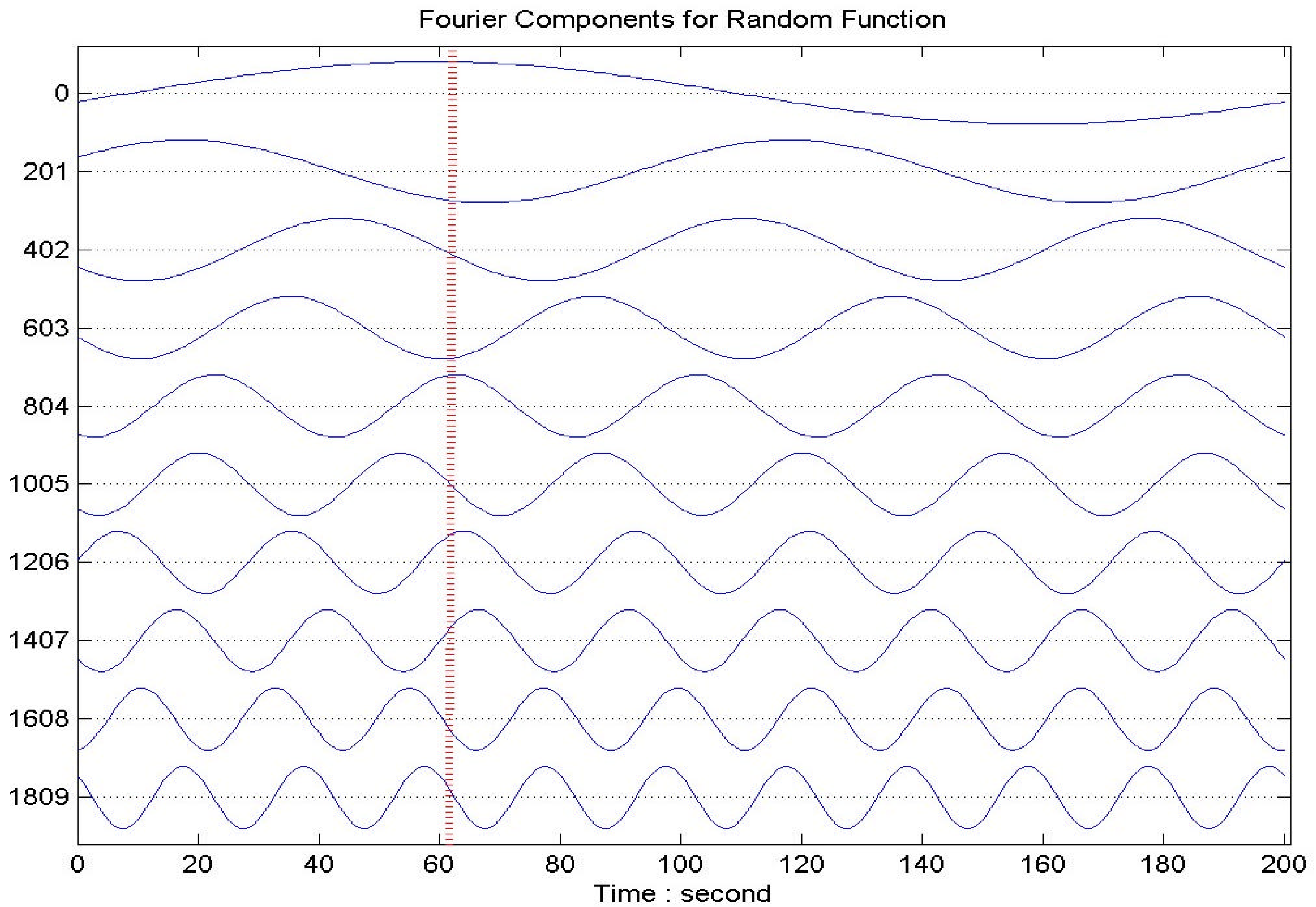
- *Any function f(t) can be expanded in terms of discrete sine or cosine functions as*

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos \omega_n t + b_n \sin \omega_n t\right).$$

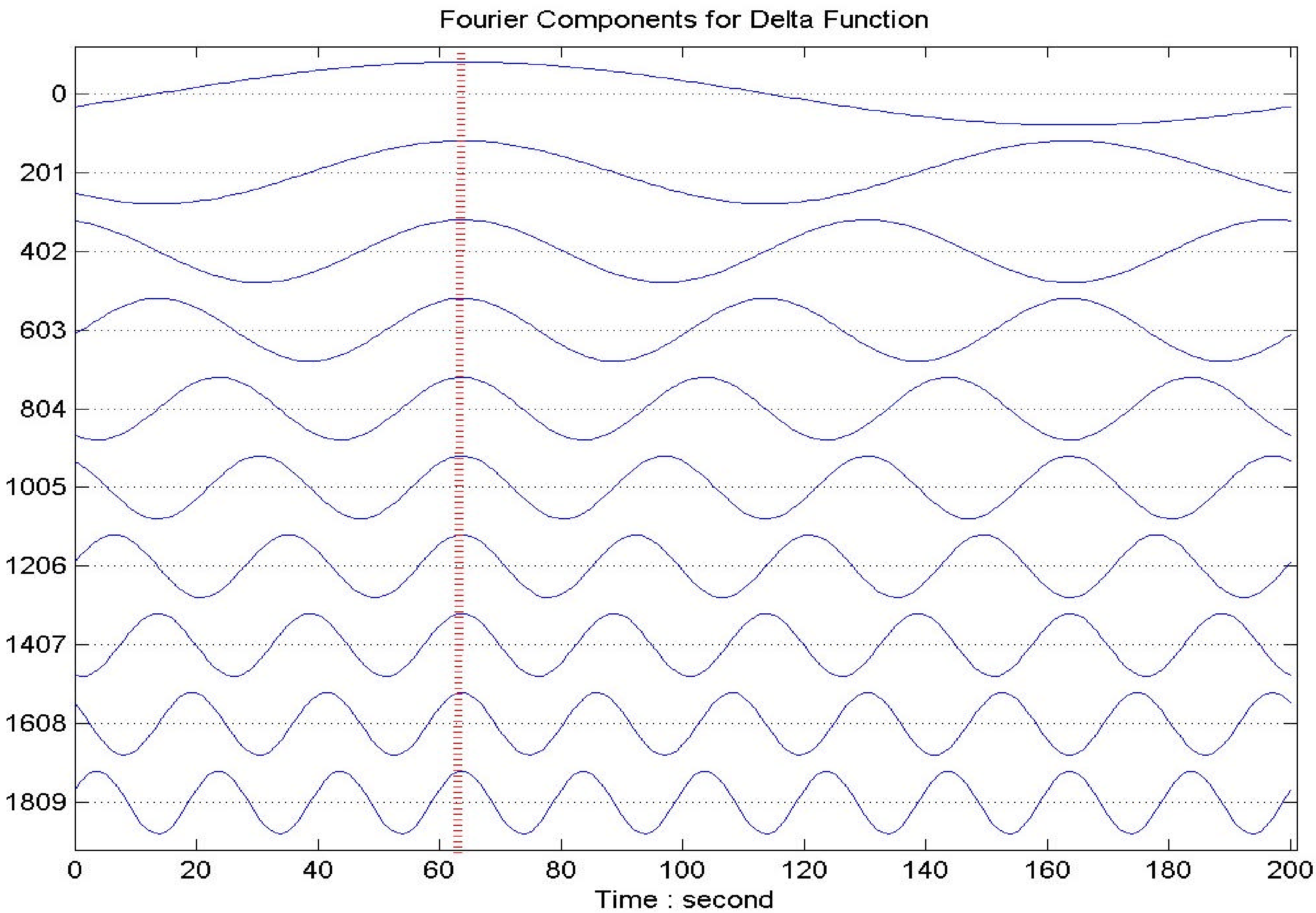
Random and Delta Functions



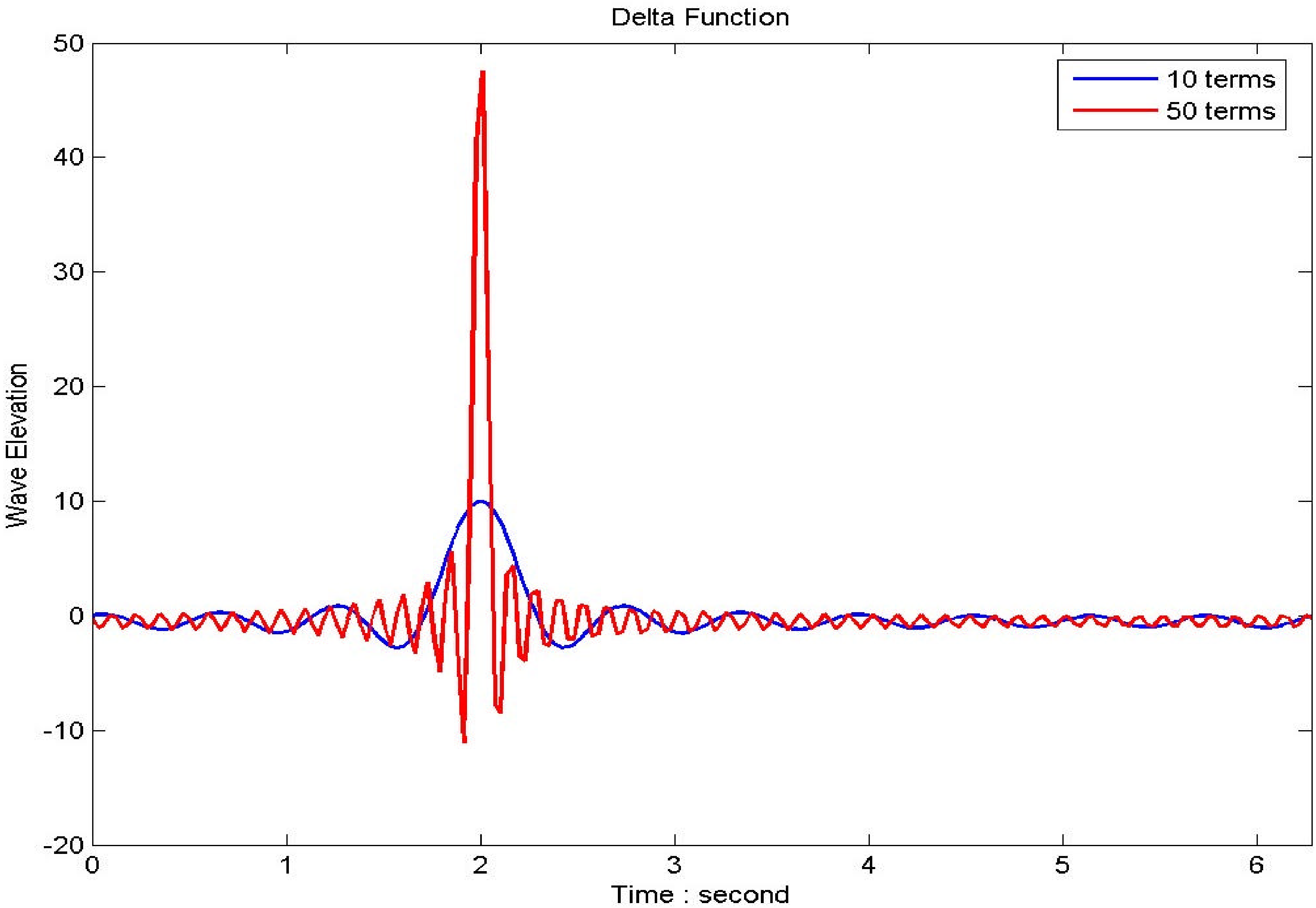
Fourier Components : Random Function



Fourier Components : Delta Function



Fourier Sums : Delta Function



Problems with Fourier Expansion

- Linear and Stationary assumptions.
  - Trigonometric function with constant frequency and amplitude over the whole time span
  - Superposition holds true limited to linear systems.
- Phase information not fully used.
  - No difference between delta and random functions in frequency spectral representation.

Data Analysis is equivalent to Information Extraction

- Data is the only connection between us and the realty.
- All our information is contained in the data.
- Data analysis is the means to extract information form the data.
- Unless we have clear understanding of the underlying processes, data analysis should not be based on *a priori* basis methods.
- Adaptive basis is the best approach to extract the maximum amount information.
- Hilbert-Huang Transform (HHT) is based on an adaptive approach.
- Data analysis is mechanical; result interpretation is the key to yield information.

The Main Data Analysis Tasks

- Distribution: global properties limited to homogeneous population only; HHT can help extract component with homogeneous scale.
- Filtering: mostly Fourier based in frequency space; HHT is a nonlinear time scale based filter.
- Regression: fit data to an *a priori* functional; HHT fits adaptively with spline.
- Correlation: need to detrend; HHT offers adaptive detrend.
- Spectral Analysis: time-frequency representation; HHT for data from nonlinear and nonstationary processes.
- Prediction: stationary processes; HHT could help here too by provide band-limited components fro easier prediction.

Motivations for a New Method

- Physical processes are mostly nonstationary
- Physical Processes are mostly nonlinear
- Data from observations are invariably too short
- Physical processes are mostly non-repeatable.

∪ Ensemble mean impossible, and temporal mean might not be meaningful for lack of ergodicity. Traditional methods inadequate.

Available Data Analysis Methods for Nonstationary (but Linear) time series

- Various probability distributions
- Spectral analysis and Spectrogram
- Wavelet Analysis
- Wigner-Ville Distributions
- Empirical Orthogonal Functions aka Singular Spectral Analysis
- Moving means
- Successive differentiations

Available Data Analysis Methods for Nonlinear (but Stationary and Deterministic) time series

- Phase space method
  - Delay reconstruction and embedding
  - Poincaré surface of section
  - Self-similarity, attractor geometry & fractals

- Nonlinear Prediction

- Lyapunov Exponents for stability

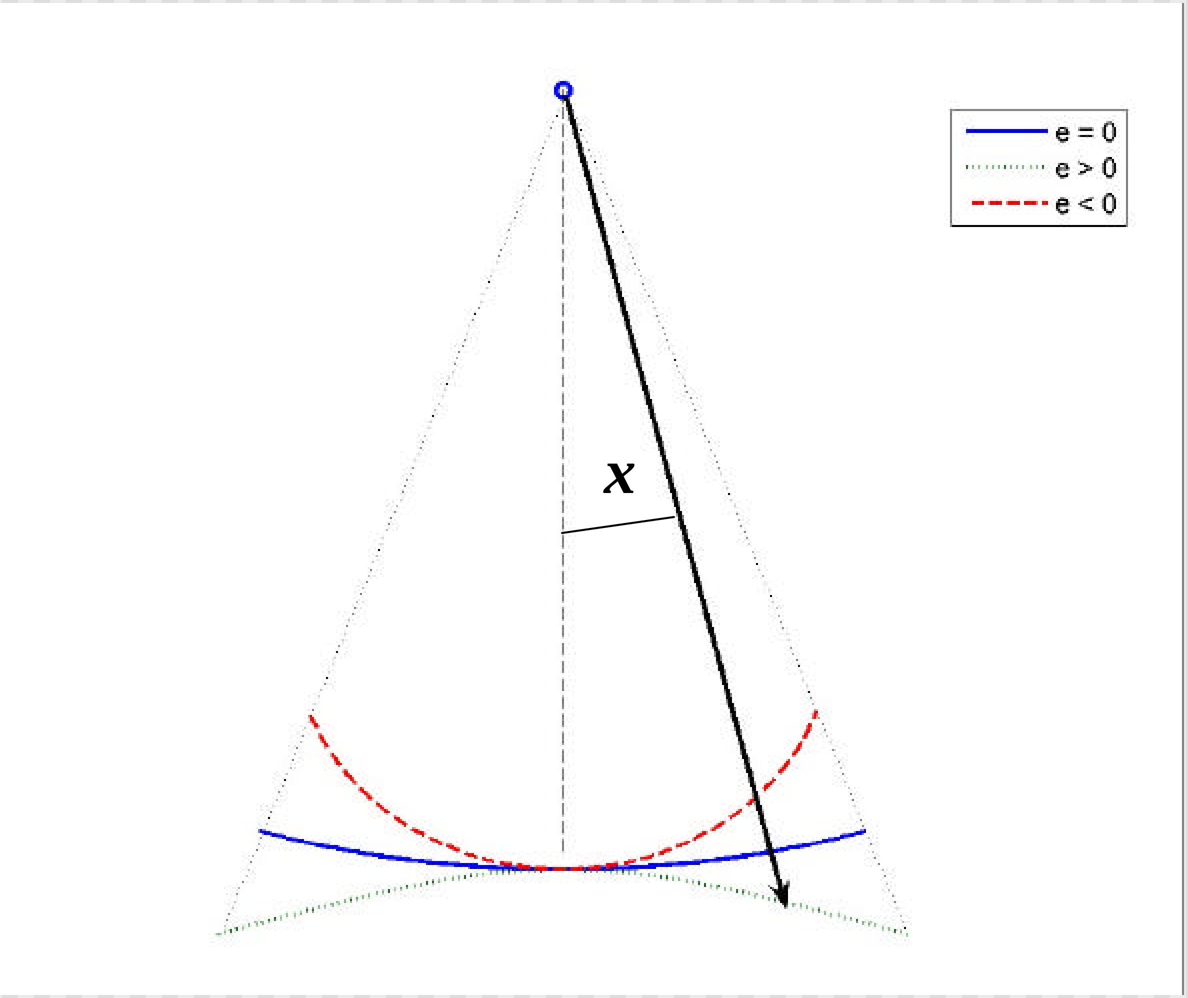
The Need for Instantaneous Frequency in Nonstationary and Nonlinear Processes

$$\frac{d^2 x}{dt^2} + x + e\, x^3 = g \cos w t$$

$$Þ \quad \frac{d^2 x}{dt^2} + x \left(1 + e\, x^2 \right) = g \cos w t$$

Þ ***Spring with position dependent constant,***
intra-wave frequency modulation;
therefore, we need instantaneous frequency.

Duffing Pendulum



$$\frac{d^2x}{dt^2} + x(1 + e\,x^2) = g\cos wt.$$

Hilbert Transform : Definition

For any $x(t) \in L^p$,

$$y(t) = \frac{1}{\pi} \wp \int_{\tau} \frac{x(\tau)}{t-\tau}\, d\tau \; ,$$

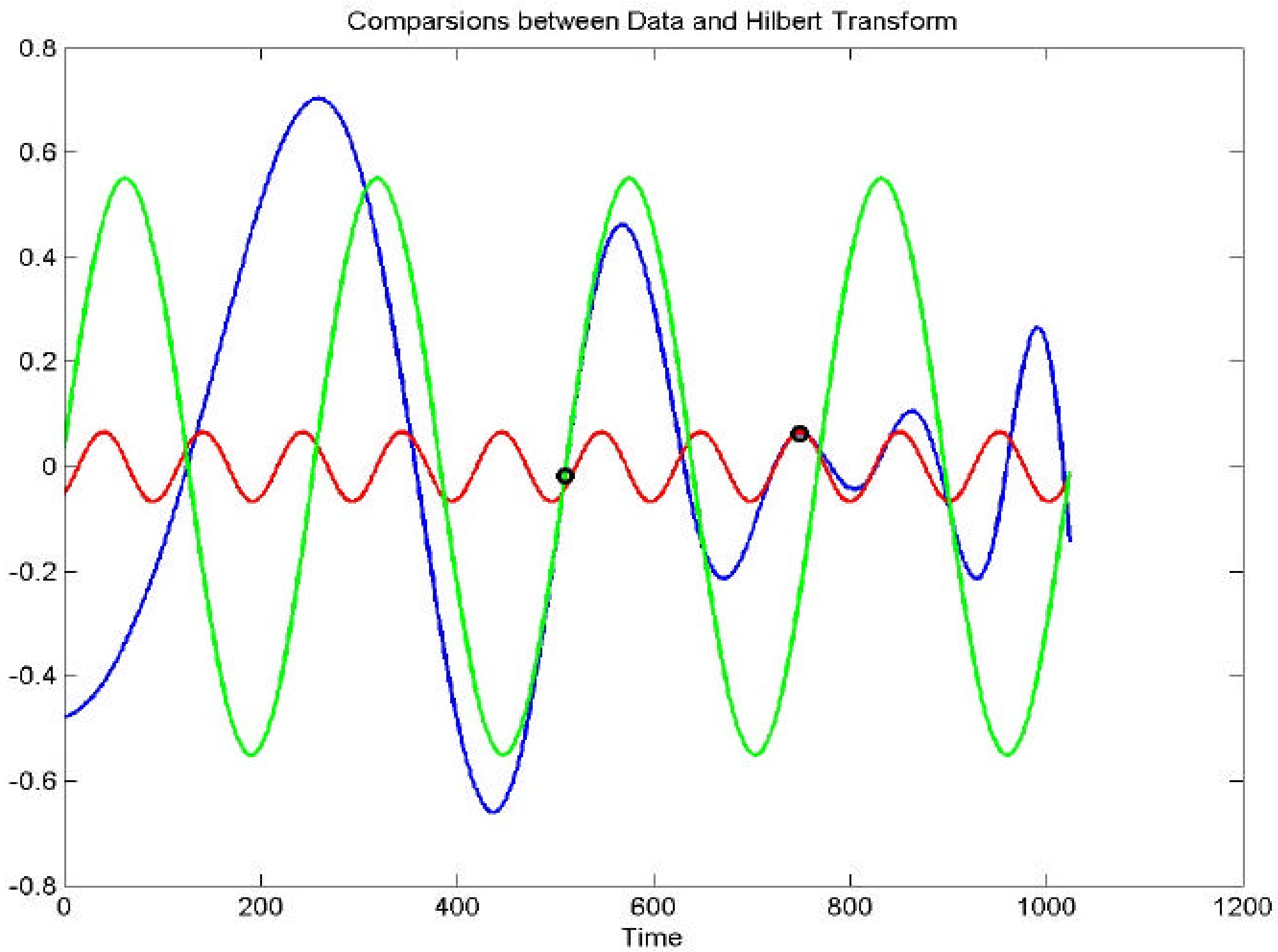
then, $x(t)$ *and* $y(t)$ *are complex conjugate :*

$$z(t) = x(t) + i\, y(t) = a(t)\, e^{i\theta(t)} ,$$

where

$$a(t) = \left(x^2 + y^2\right)^{1/2} \text{ and } \theta(t) = \tan^{-1}\frac{y(t)}{x(t)} .$$

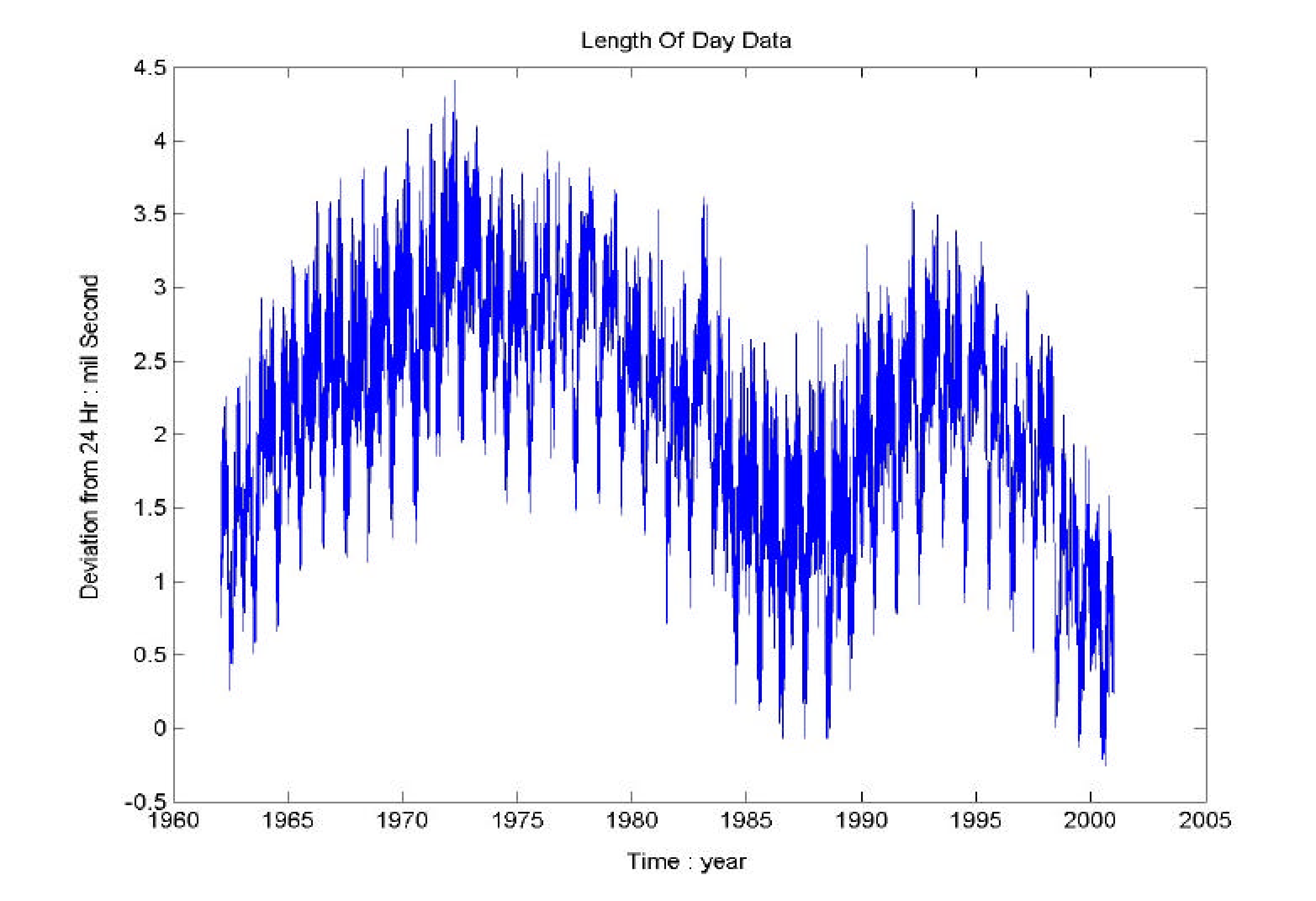
Hilbert Transform Fit



The Traditional View of the Hilbert Transform for Data Analysis

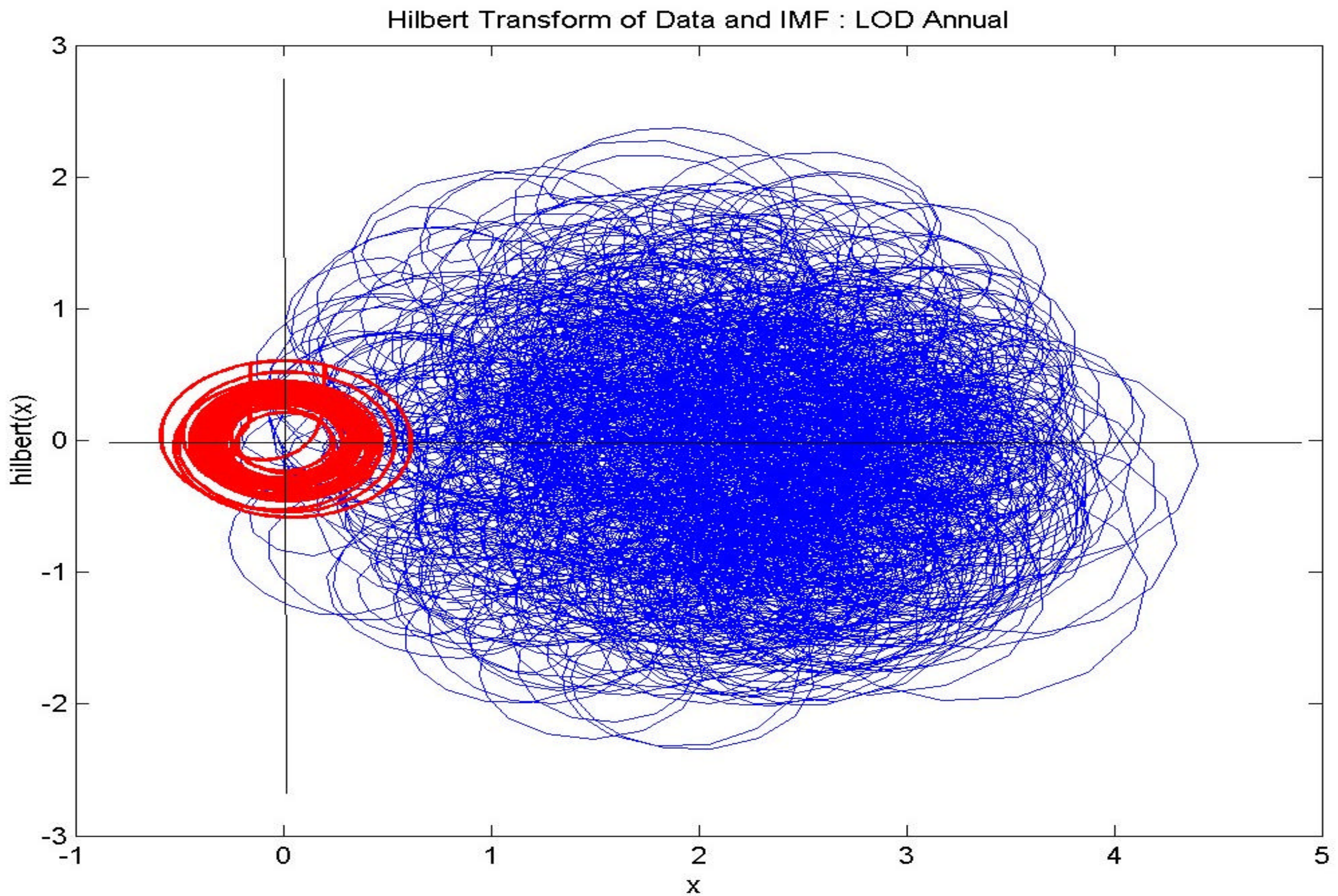
Traditional View

a la Hahn (1995) : Data LOD



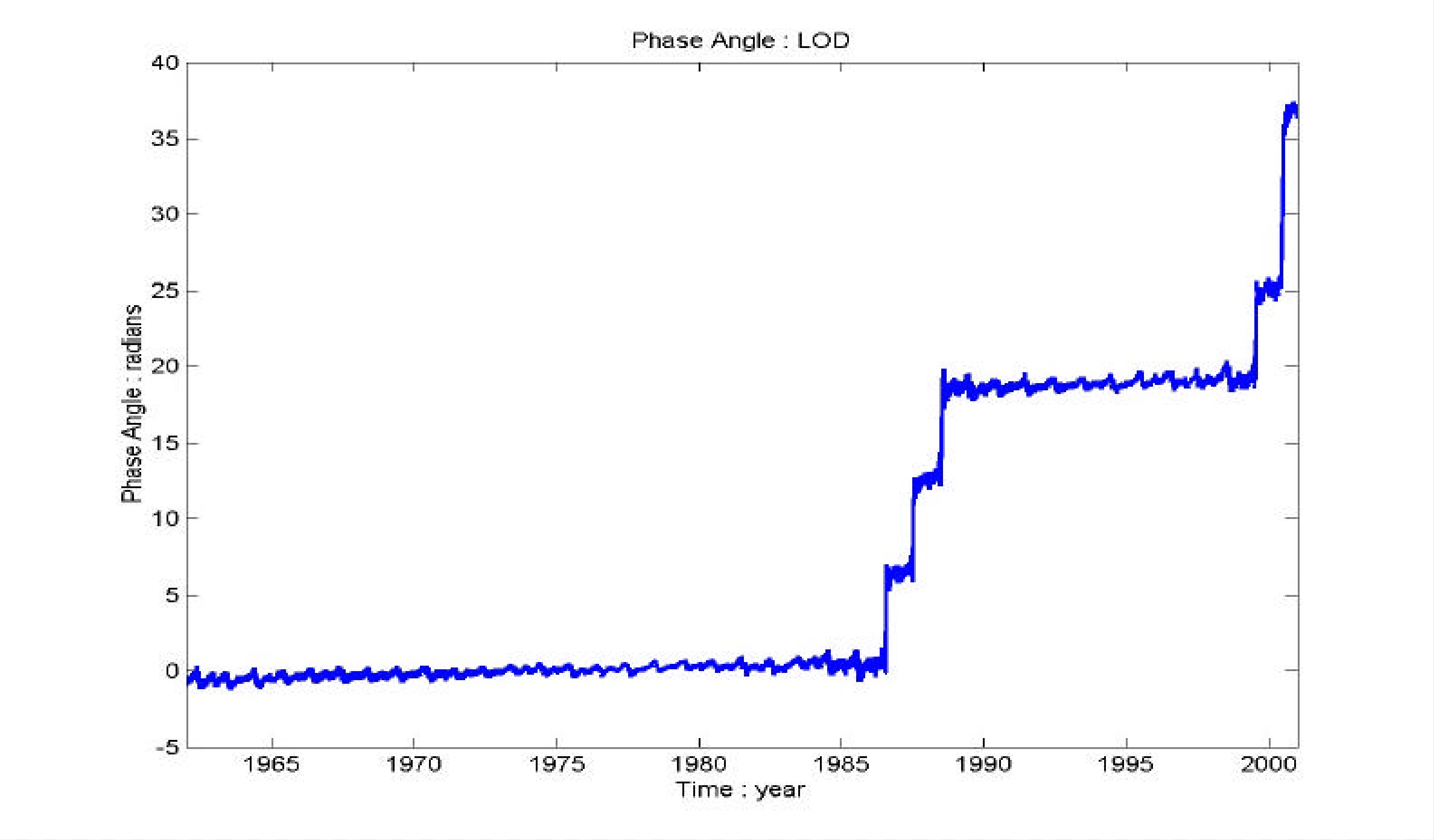
Traditional View

a la Hahn (1995) : Hilbert



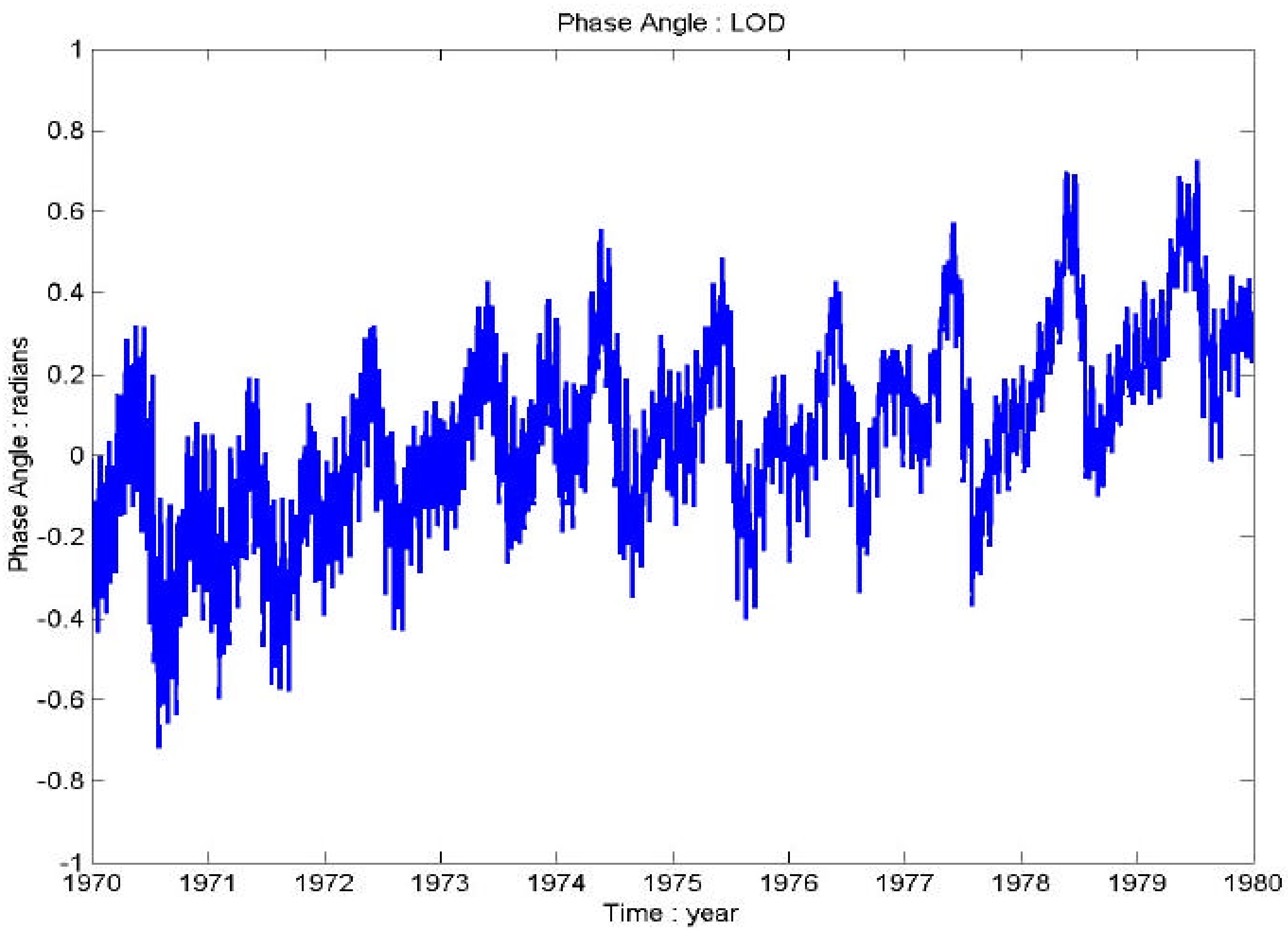
Traditional View

a la Hahn (1995) : Phase Angle



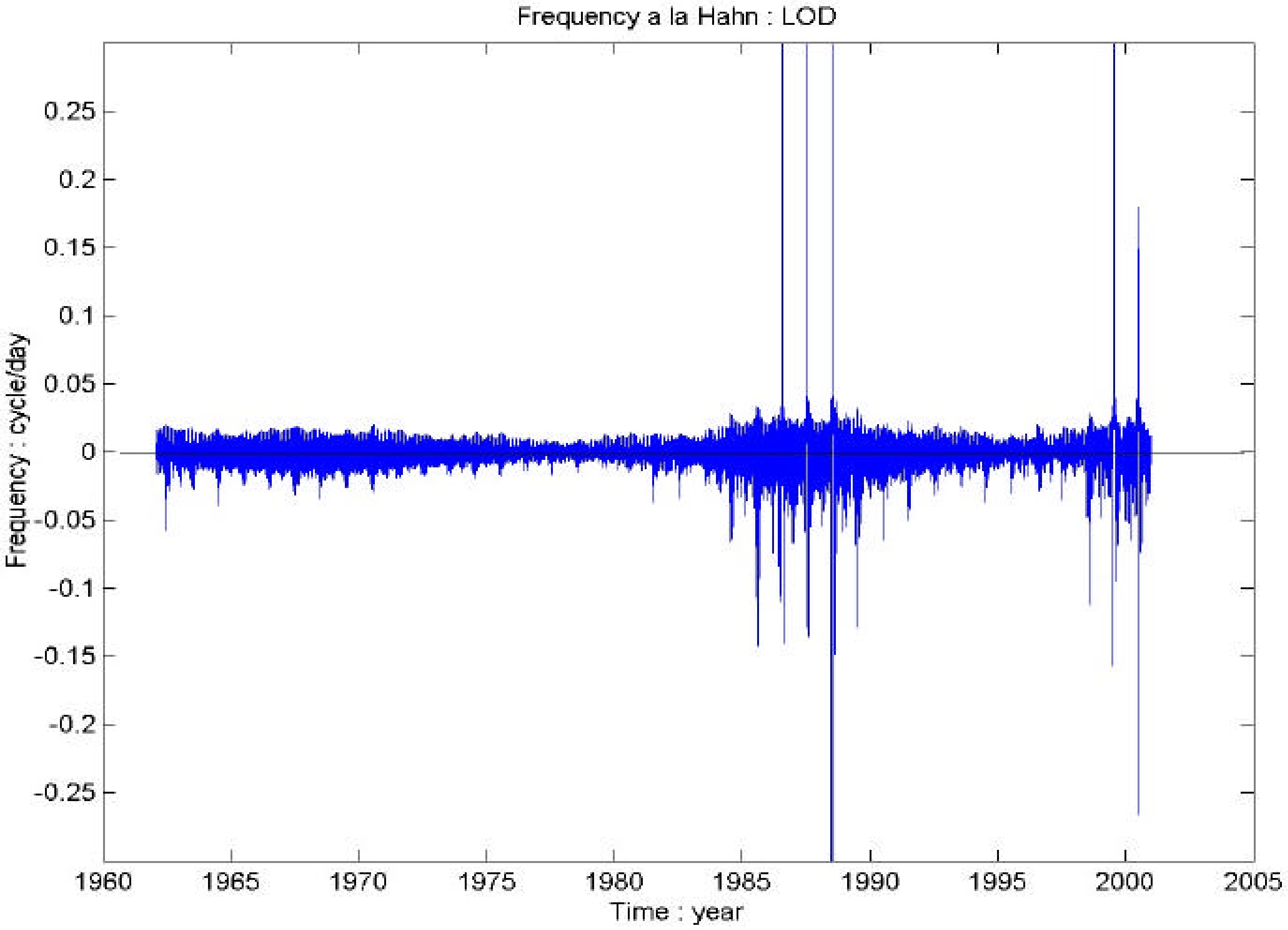
Traditional View

a la Hahn (1995) : Phase Angle Details



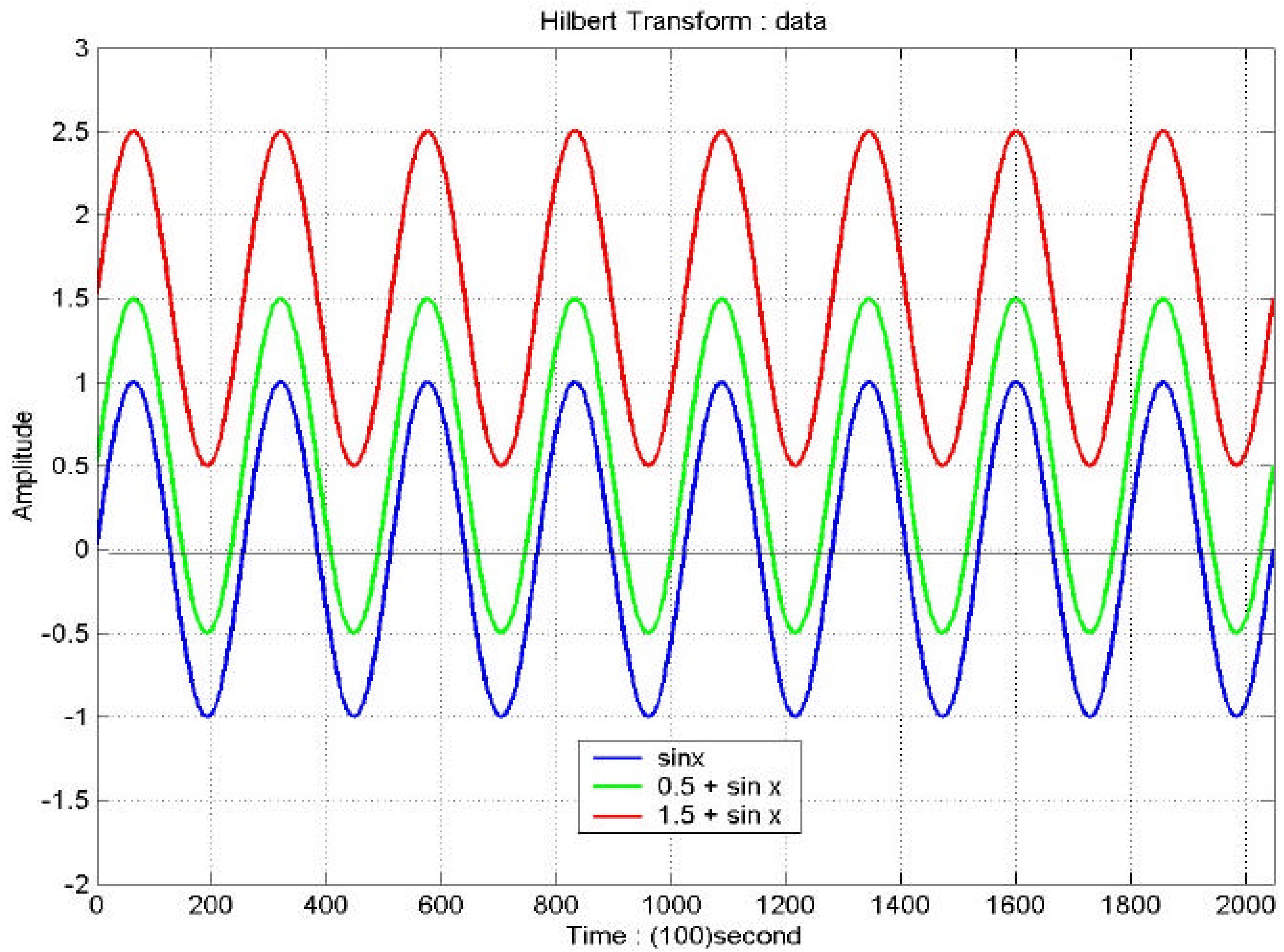
Traditional View

a la Hahn (1995) : Frequency



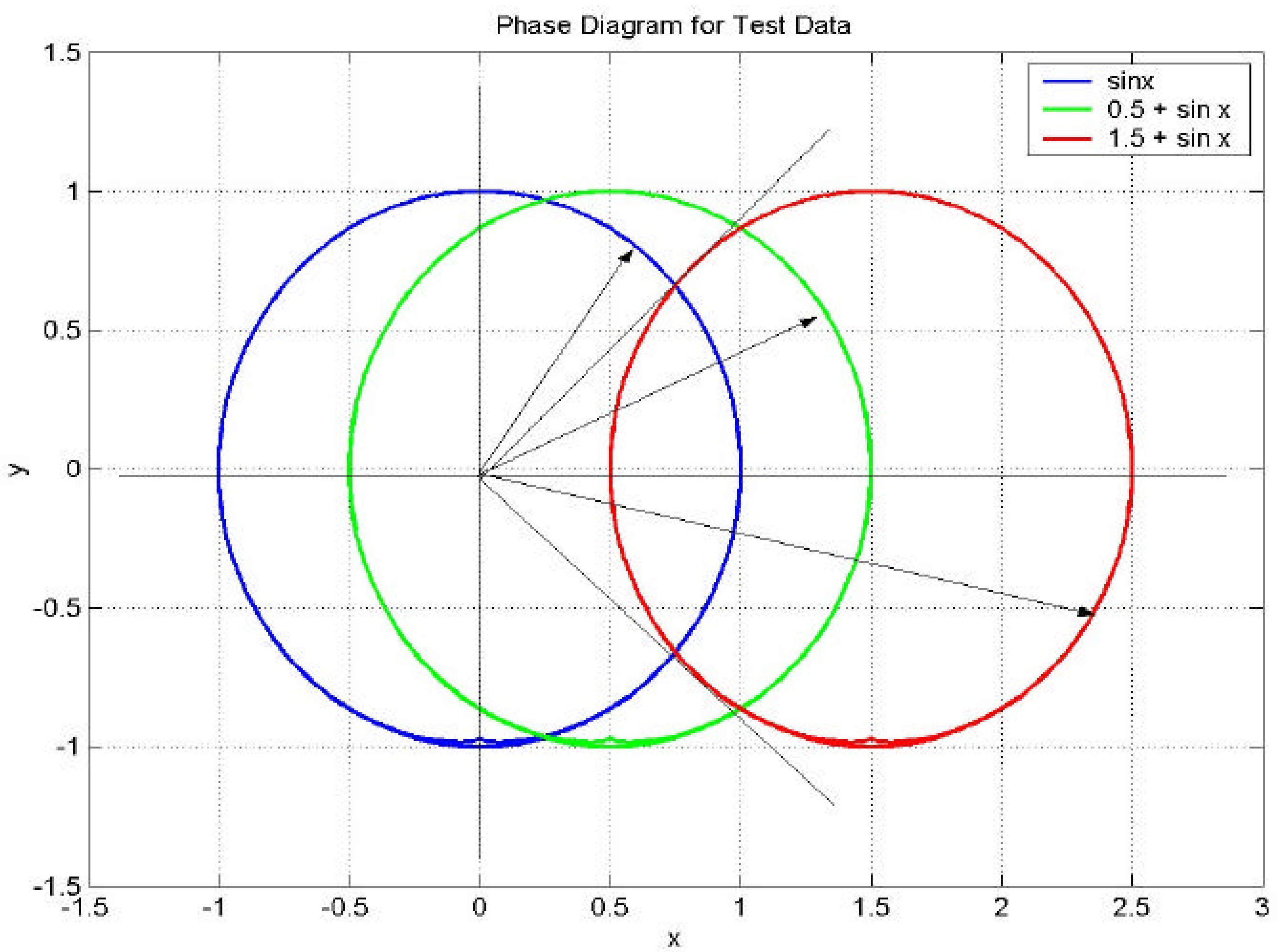
Why the traditional view does not work?

Hilbert Transform $a \cos \theta + b$: Data



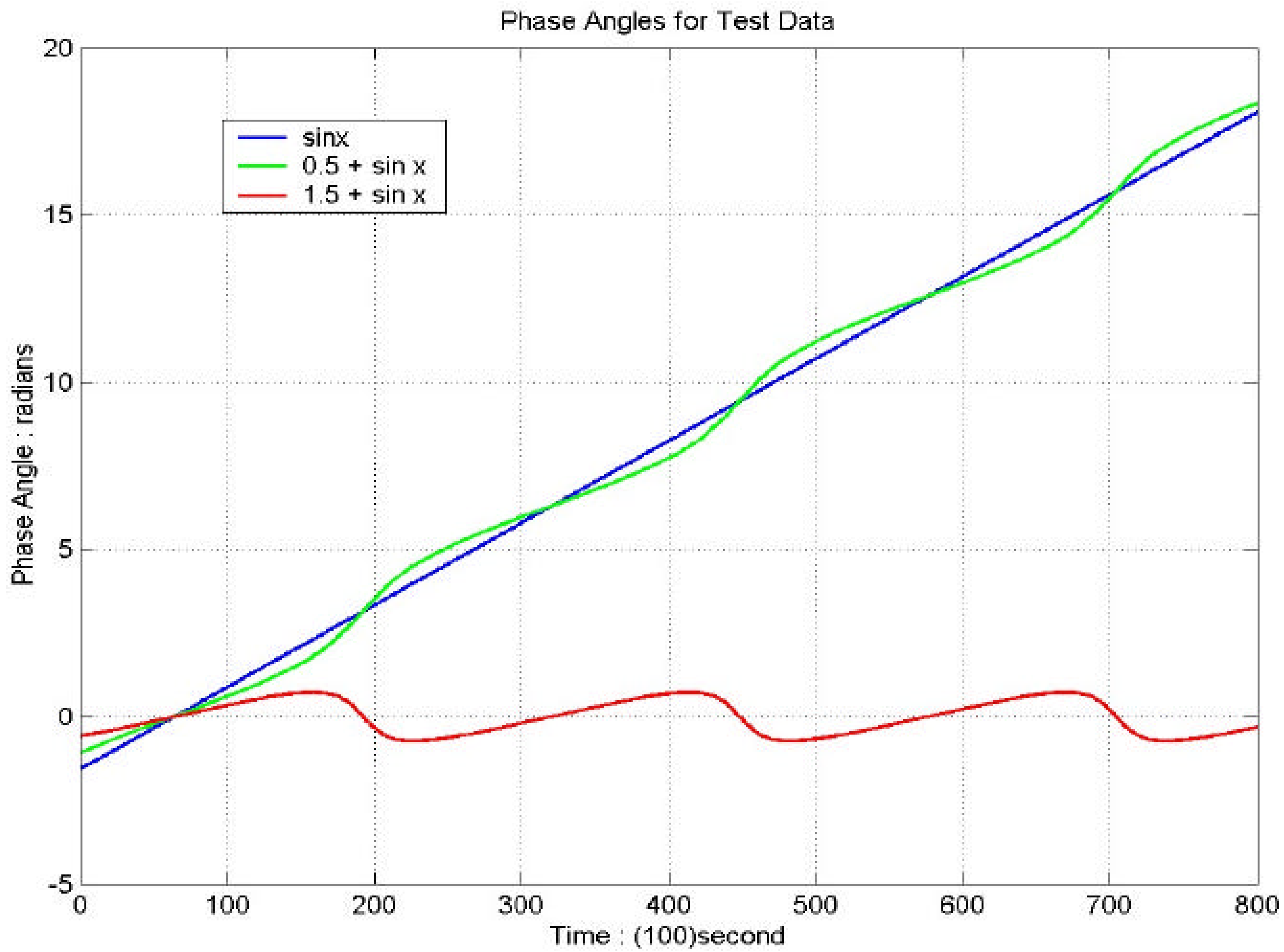
Hilbert Transform $a \cos$ ❑ + b :

Phase Diagram

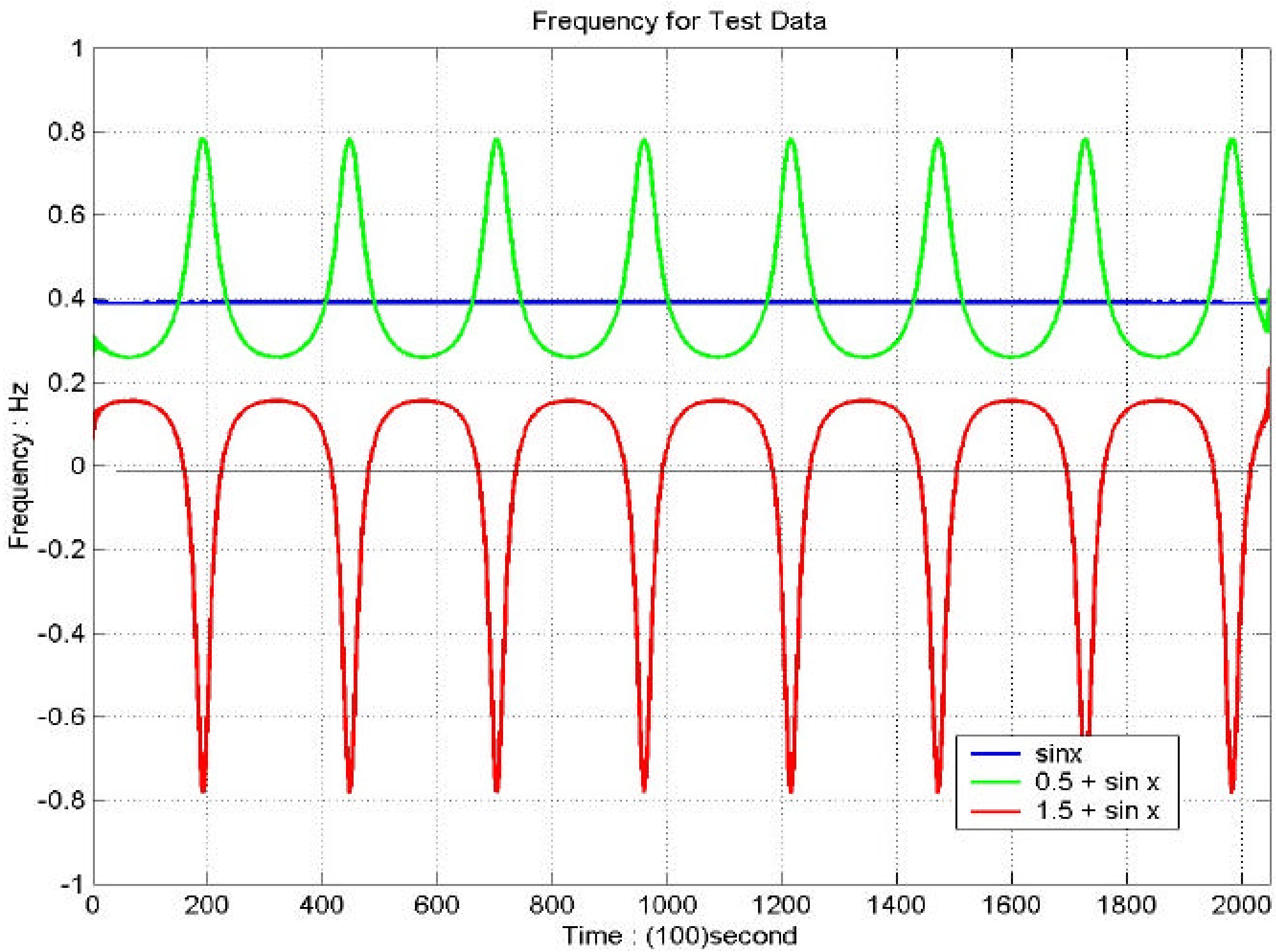


Hilbert Transform *a cos* ❑ + *b* :

Phase Angle Details



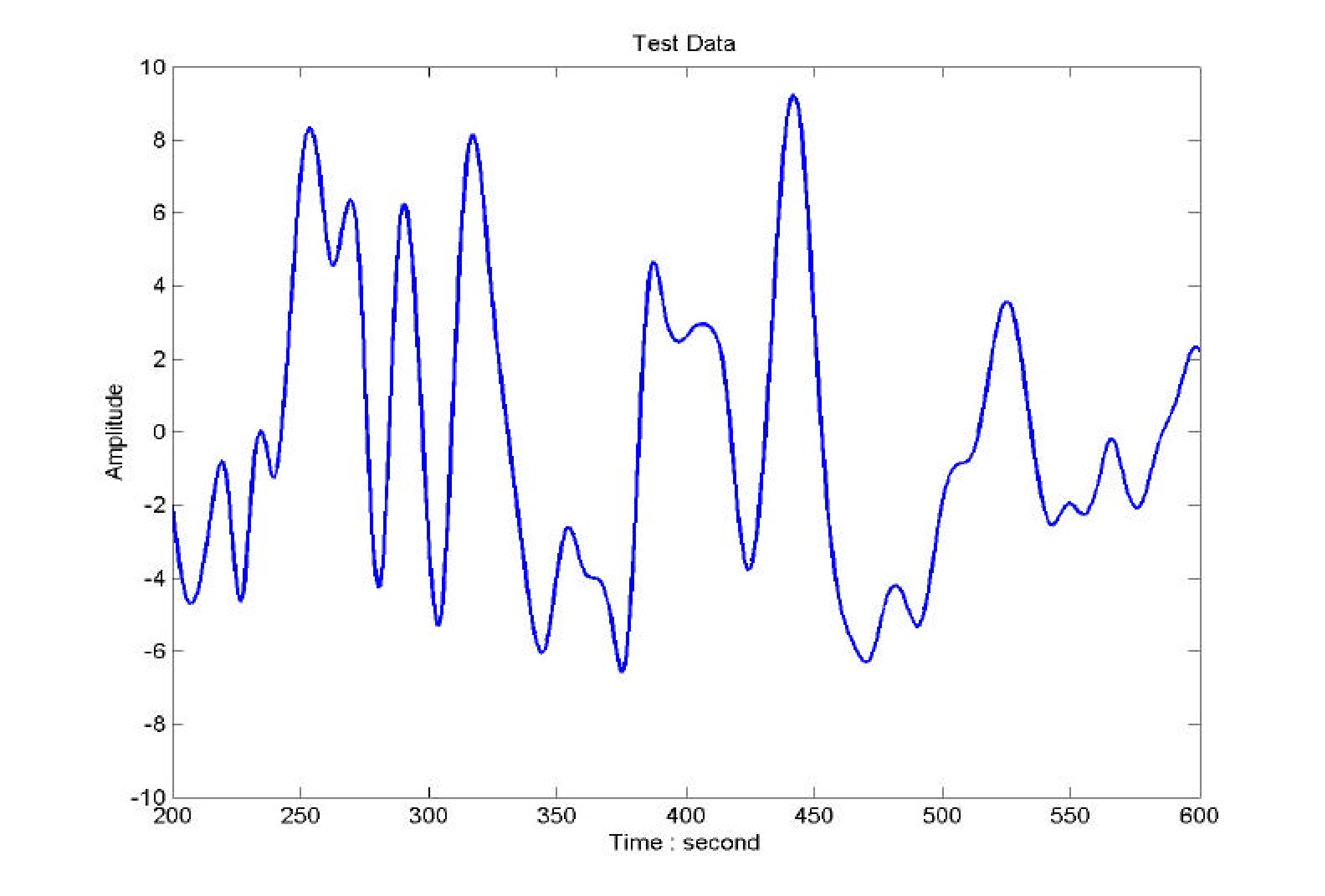
Hilbert Transform *a cos* ❑ + *b* *:*
Frequency



The Empirical Mode Decomposition Method and Hilbert Spectral Analysis

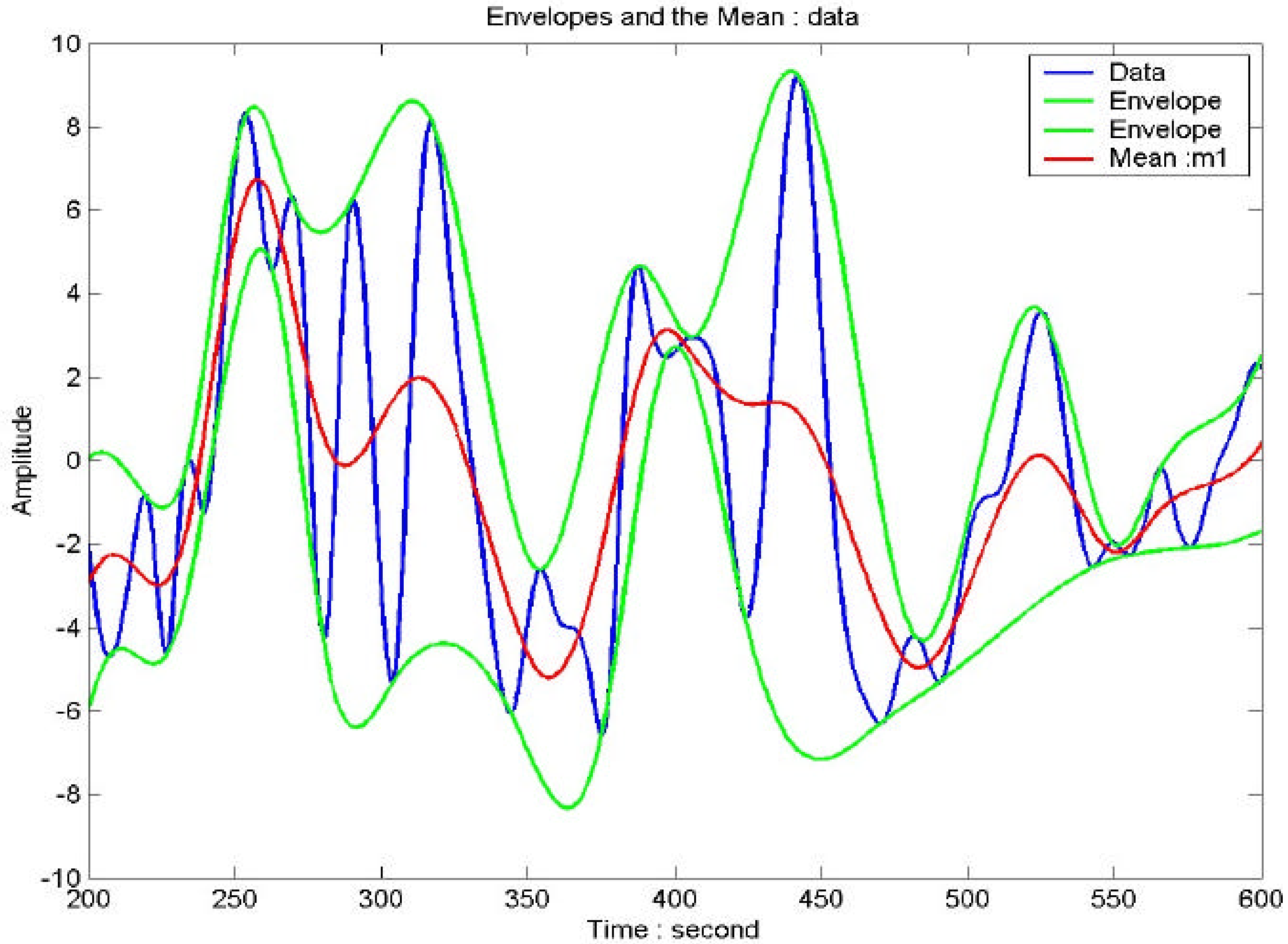
Sifting

Empirical Mode Decomposition:
Methodology : Test Data

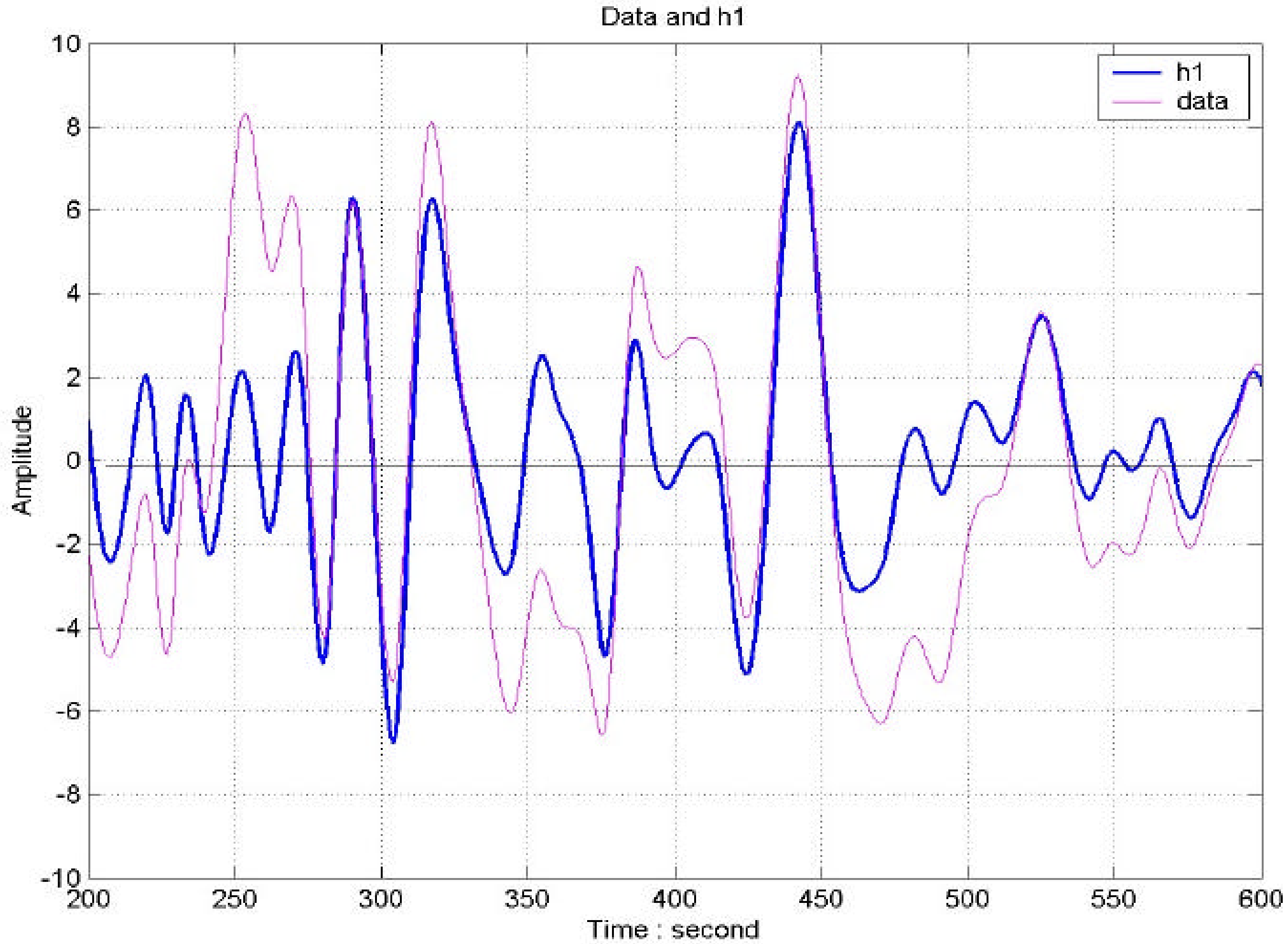


Empirical Mode Decomposition:

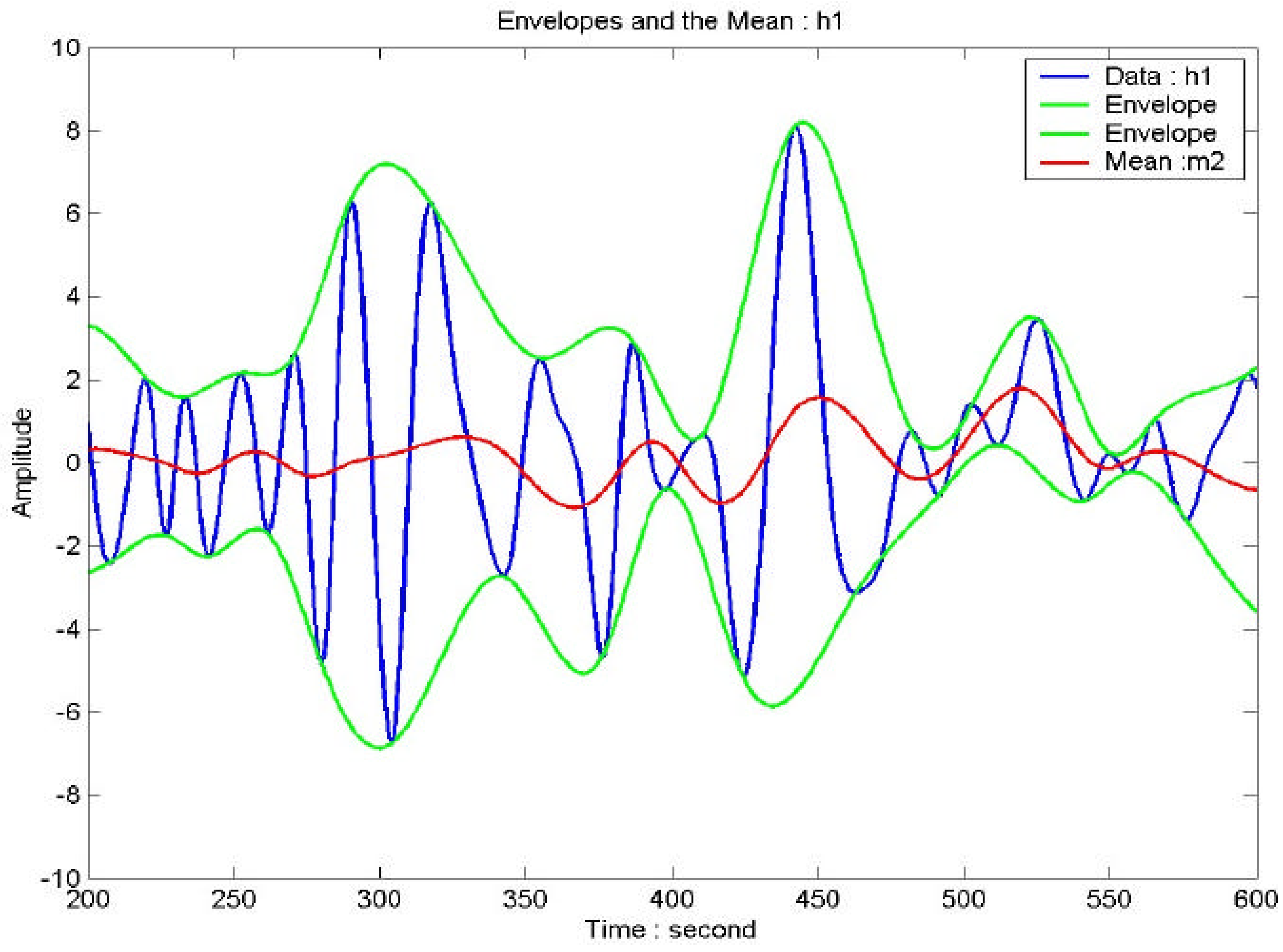
Methodology : data and m1



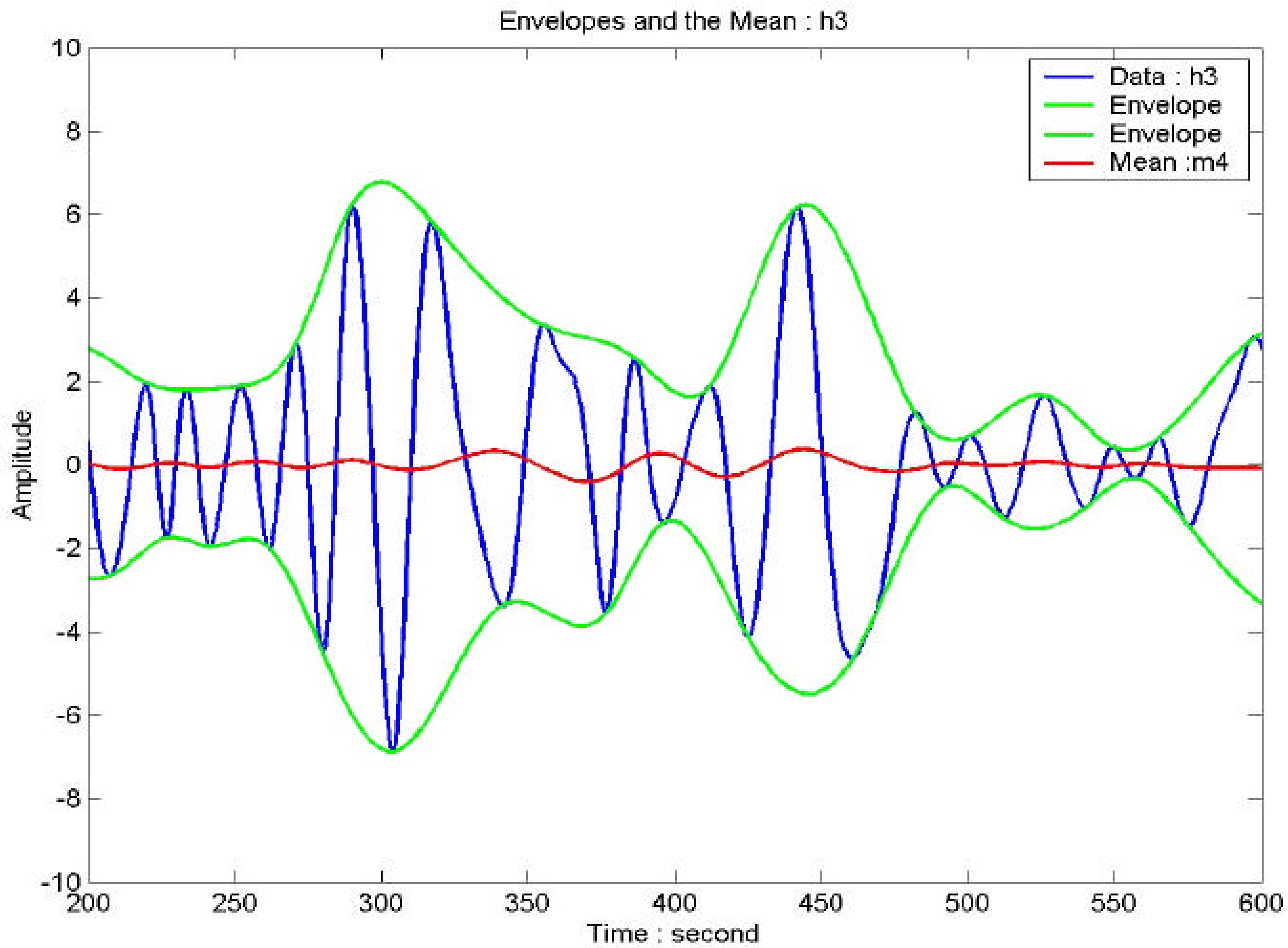
Empirical Mode Decomposition:
Methodology : data & h1



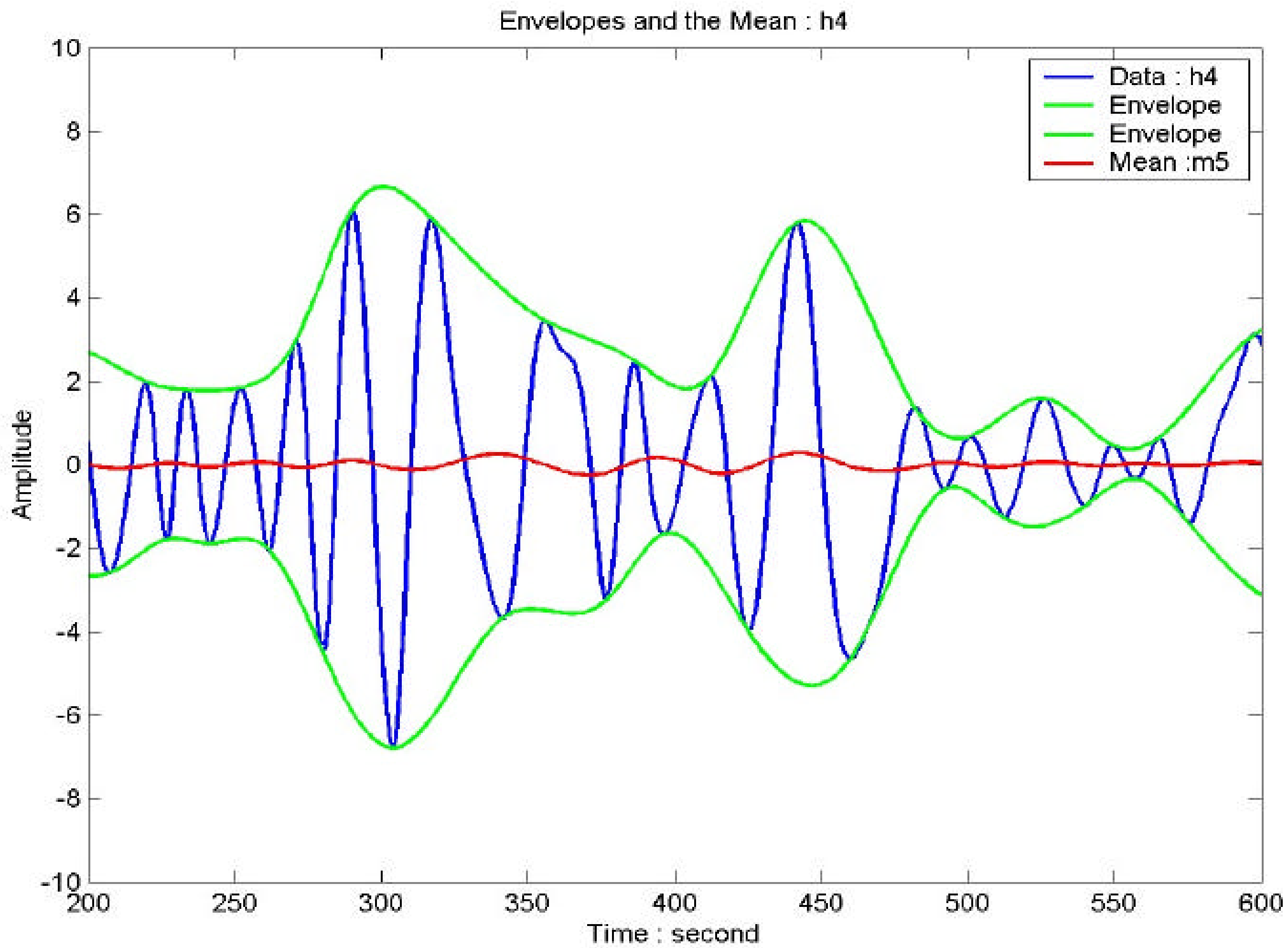
Empirical Mode Decomposition:
Methodology : h1 & m2



Empirical Mode Decomposition:
Methodology : h3 & m4



Empirical Mode Decomposition:
Methodology : h4 & m5



Empirical Mode Decomposition

Sifting : to get one IMF component

$$x(t) - m_1 = h_1 \,,$$

$$h_1 - m_2 = h_2 \,,$$

$$\cdots\cdots$$

$$\cdots\cdots$$

$$h_{k-1} - m_k = h_k \,.$$

$$\Rightarrow \quad h_k = c_1 \,.$$

Two Stoppage Criteria : S and SD

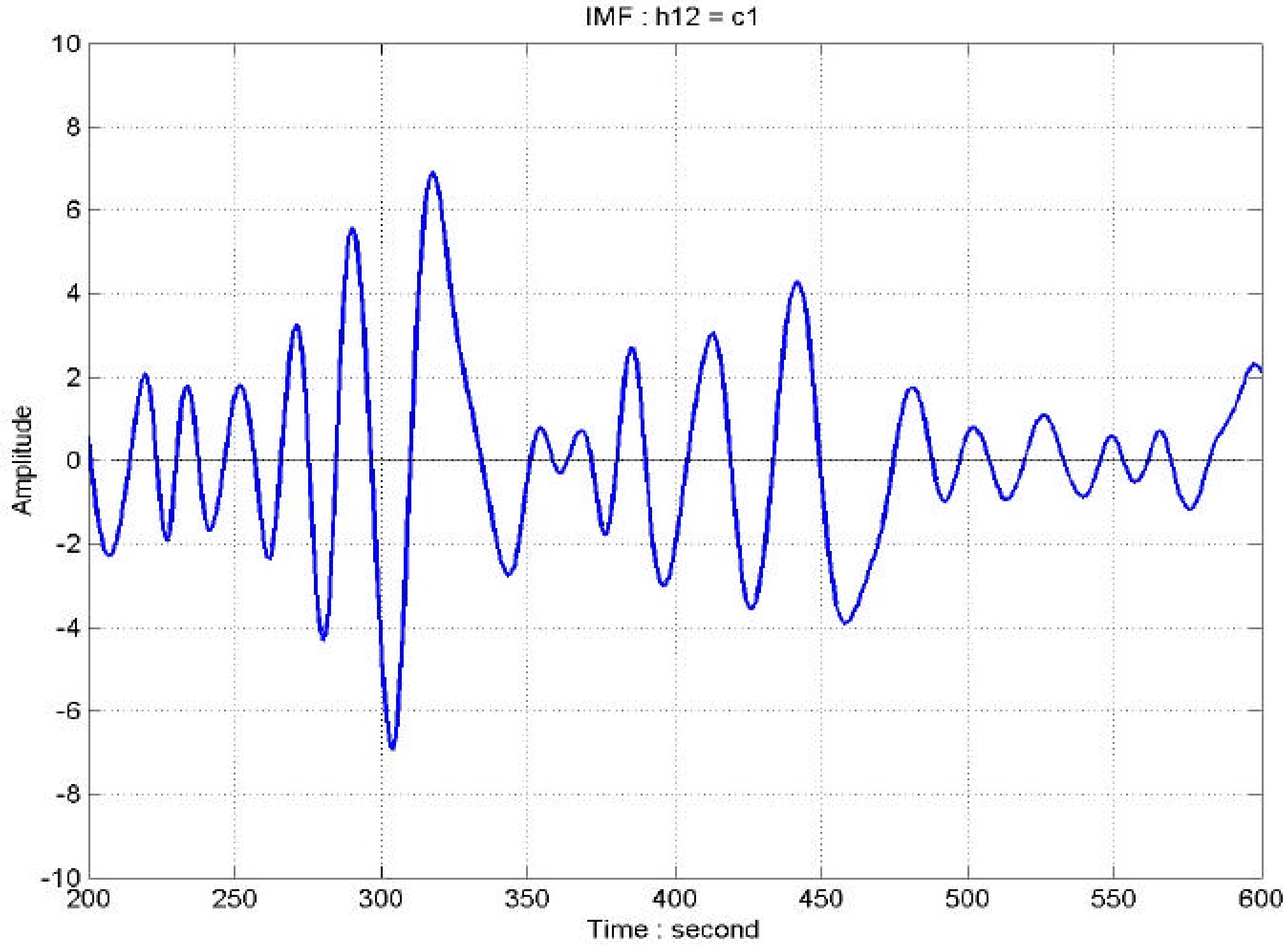
A. The S number : S is defined as the consecutive number of siftings, in which the numbers of zero-crossing and extrema are the same for these S siftings.

B. SD is small than a pre-set value, where

$$SD = \sum_{t=0}^{T} \frac{\left| h_{k-1}(t) - h_k(t) \right|^2}{h_{k-1}^2(t)} .$$

Empirical Mode Decomposition:

Methodology : IMF c1



Definition of the Intrinsic Mode Function (IMF)

Any function having the same numbers of zero - cros sin gs and extrema, and also having symmetric envelopesdefined by local maxima and min ima respectively isdefined asan Intrinsic ModeFunction(IMF).

All IMF enjoys good HilbertTransform :

Þ Þ $c(t) = a(t)\,e^{i\theta(t)}$

Empirical Mode Decomposition

Sifting : to get all the IMF components

$$x(t) - c_1 = r_1 \ ,$$

$$r_1 - c_2 = r_2 \ ,$$

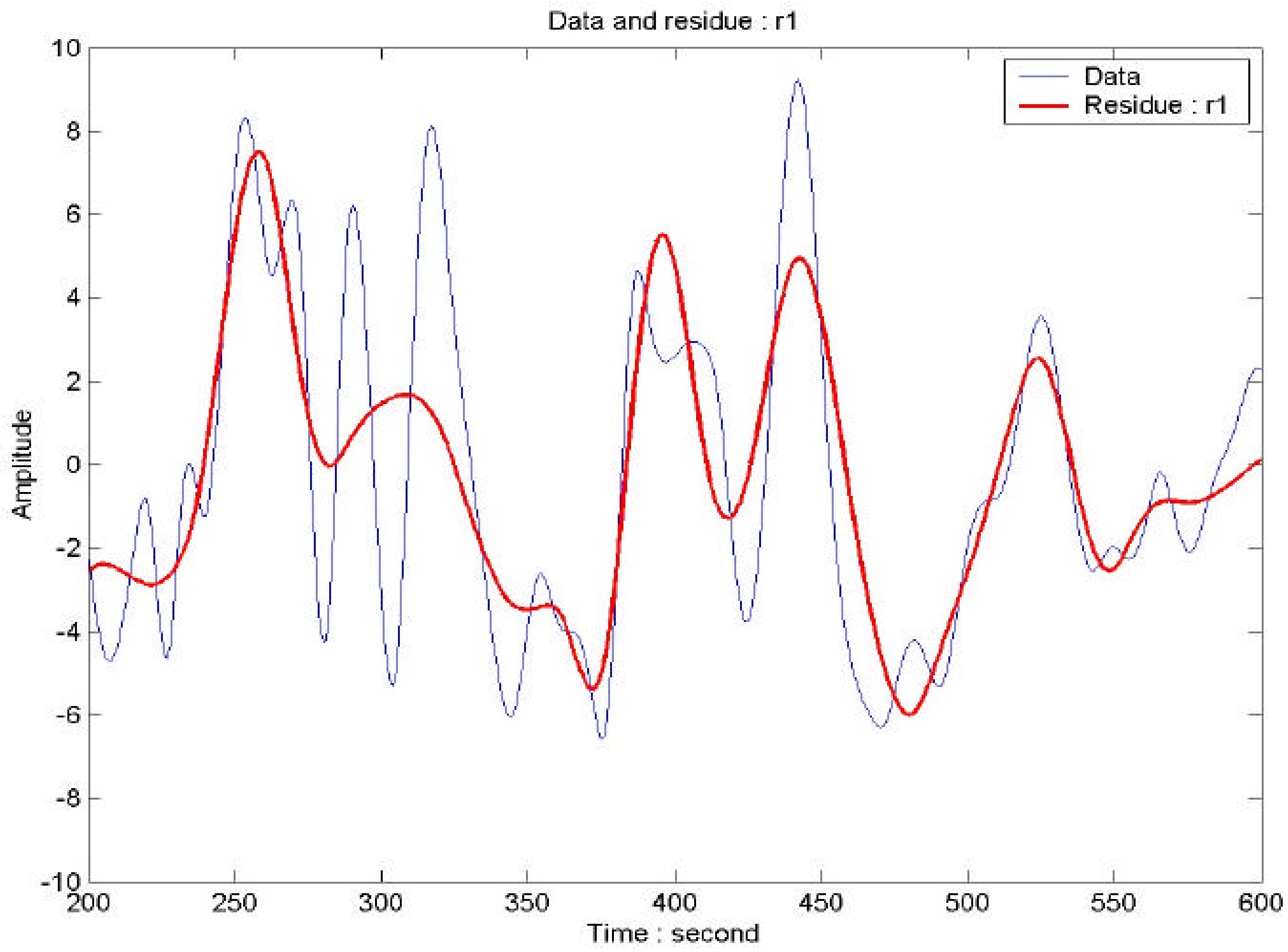
$$\cdots$$

$$r_{n-1} - c_n = r_n \ .$$

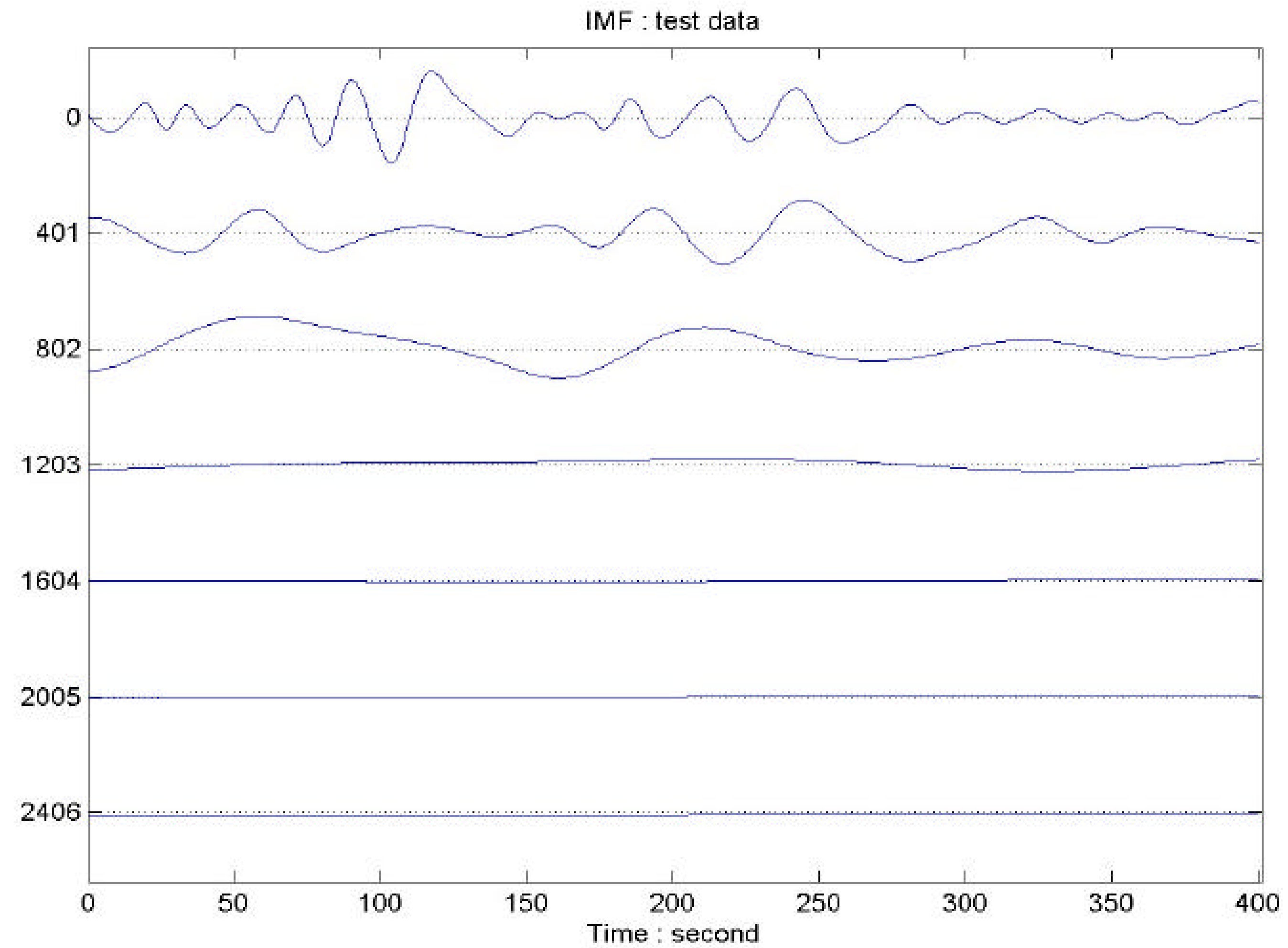
$$\Rightarrow \ x(t) - \sum_{j=1}^{n} c_j = r_n \ .$$

Empirical Mode Decomposition:

Methodology : data & r1



Empirical Mode Decomposition:
Methodology : IMFs



Definition of Instantaneous Frequency

The FourierTransform of the Instrinsic Mode Funnction, c(t), gives

$$W(w) = \int_t a(t)\, e^{i(q - wt)}\, dt$$

By Stationary phaseapproximation we have

$$\frac{dq(t)}{dt} = w \ ,$$

This isdefined asthe Ins tan tan eous Frequency .

Comparison between FFT and HHT

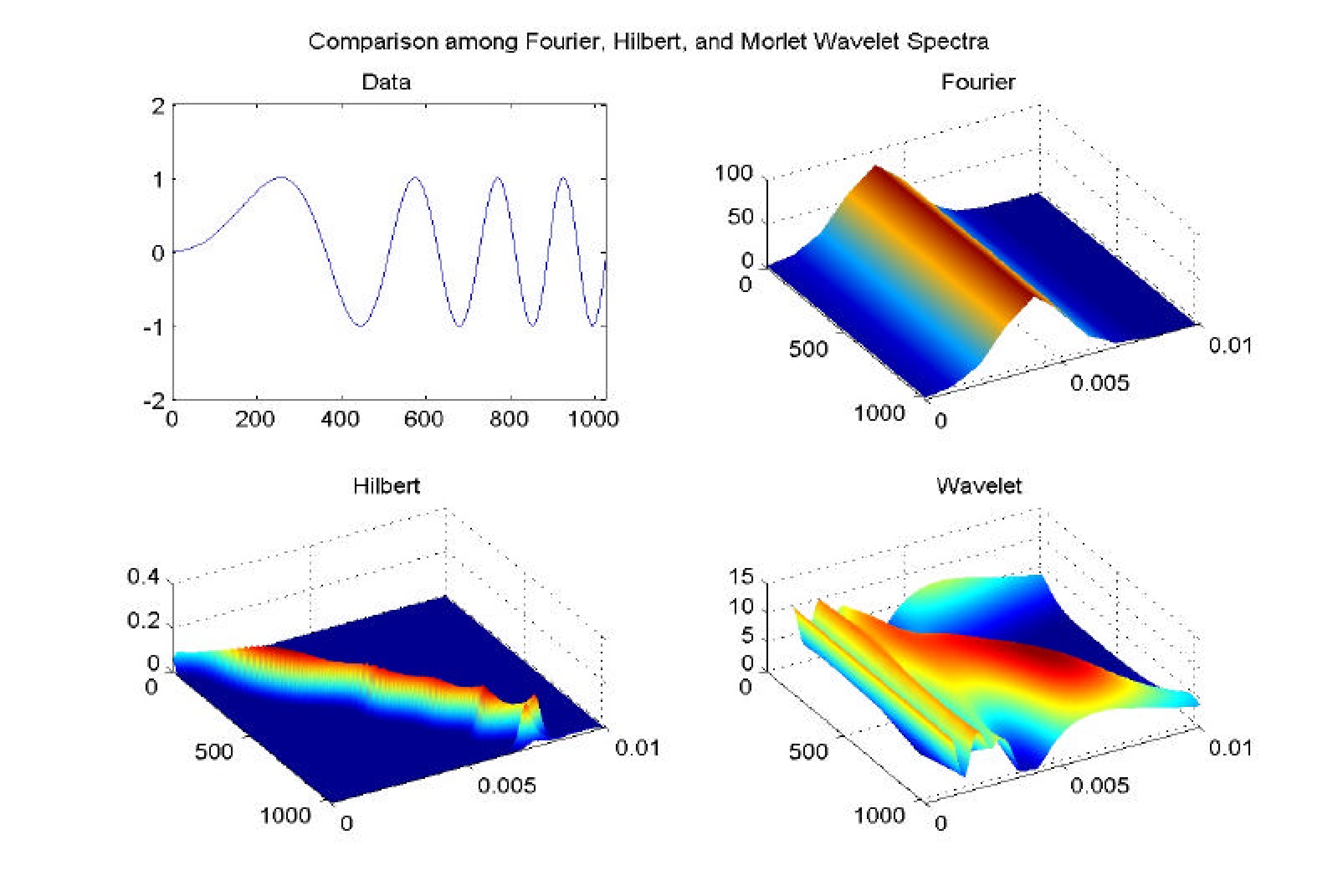
1. FFT :

$$x(t) = \Re \sum_j a_j e^{i\omega_j t}.$$

2. HHT :

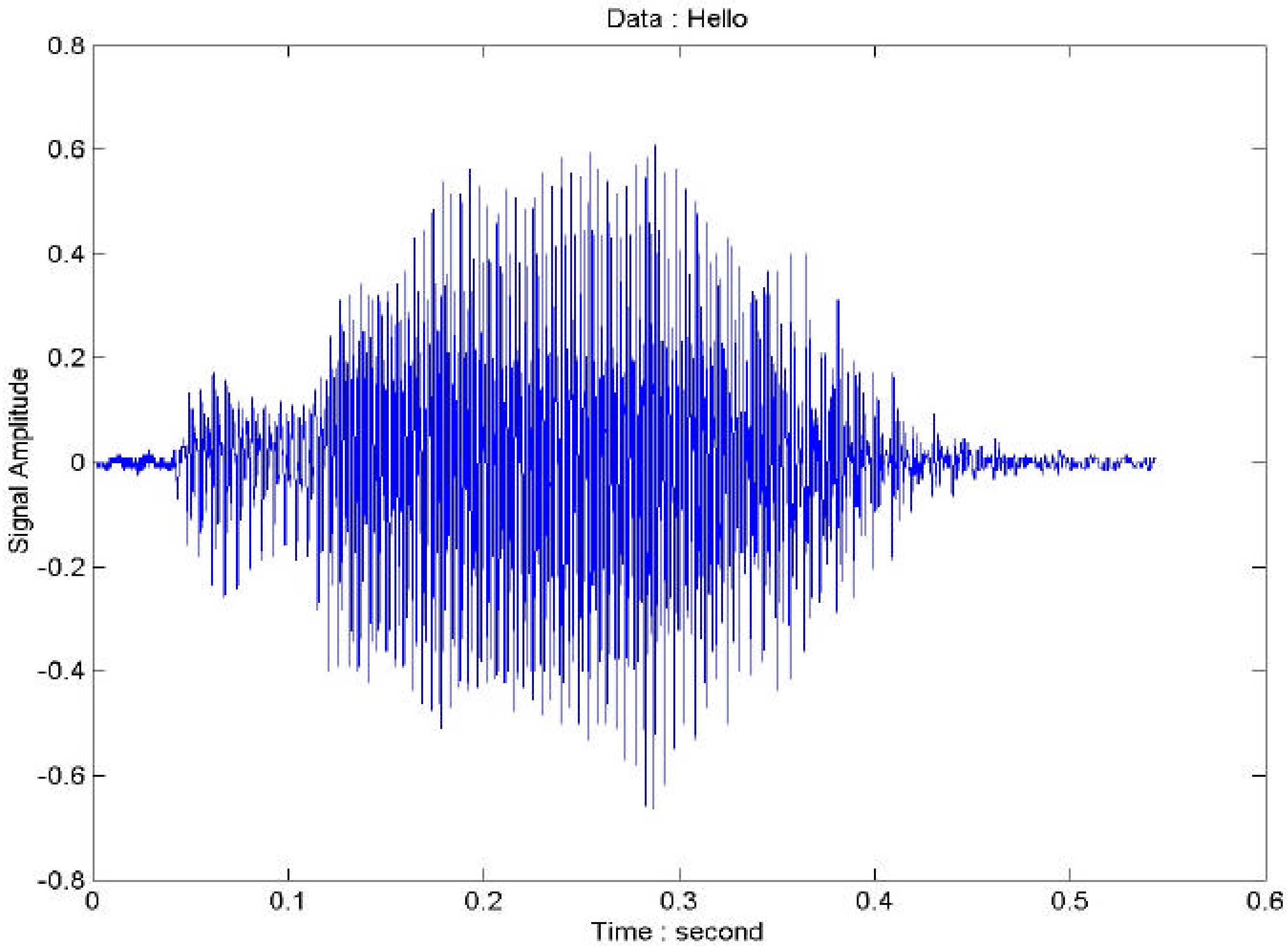
$$x(t) = \Re \sum_j a_j(t)\, e^{i\int \omega_j(t)\,dt}.$$

Comparisons: Fourier, Hilbert & Wavelet

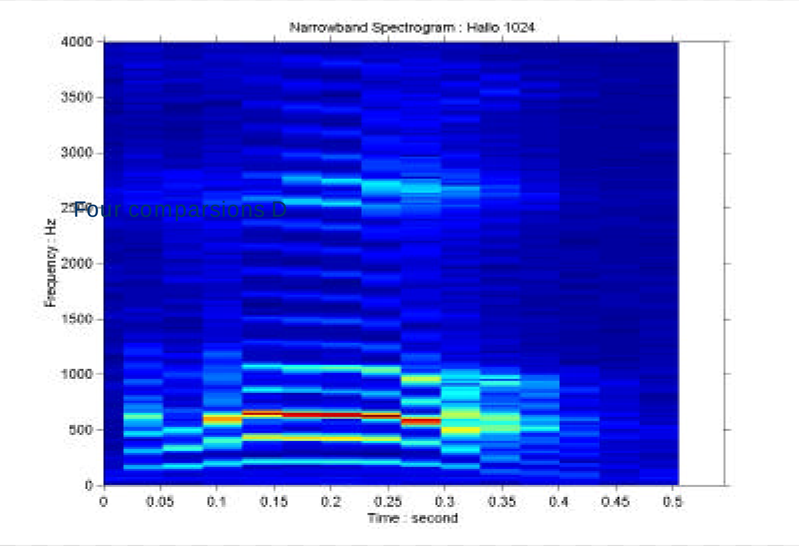


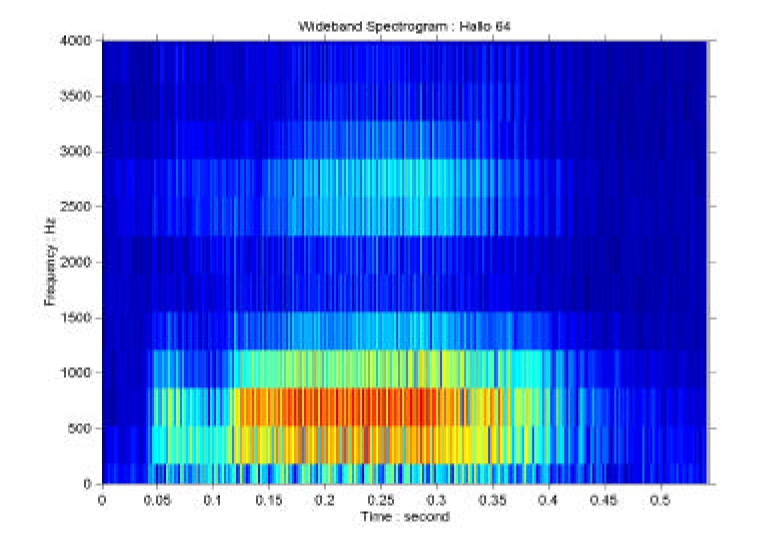
Speech Analysis

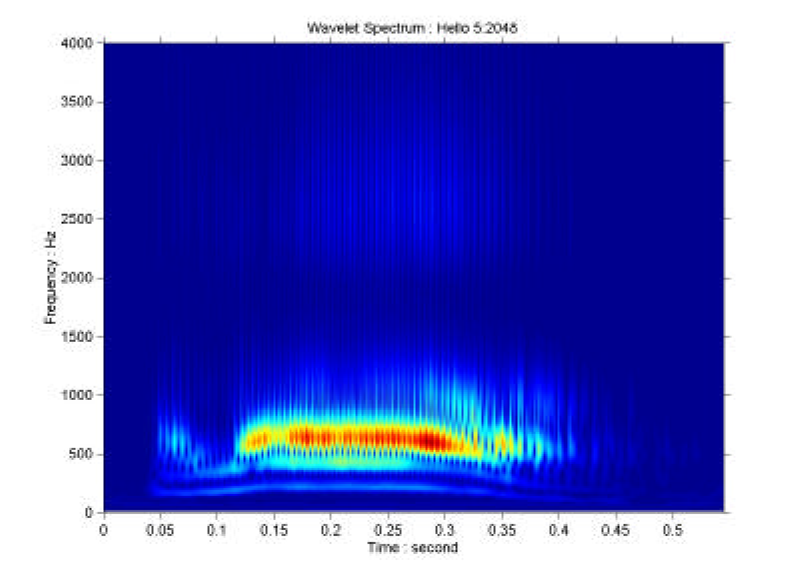
Hello : Data

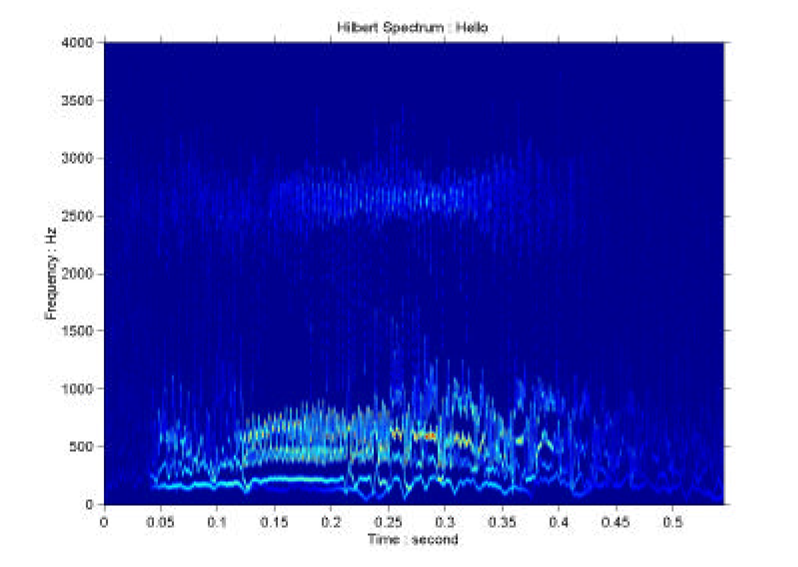


Four comparsions D



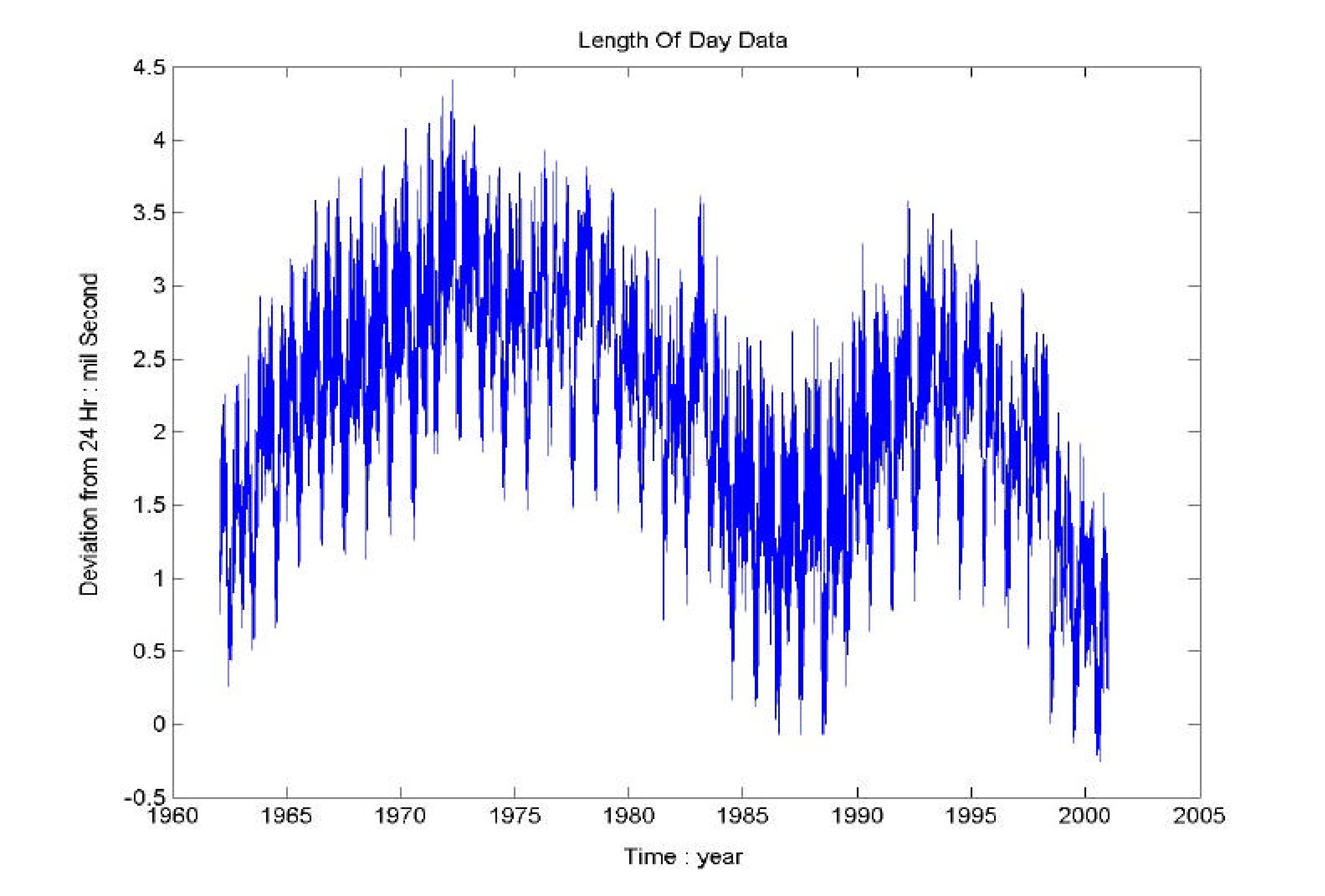




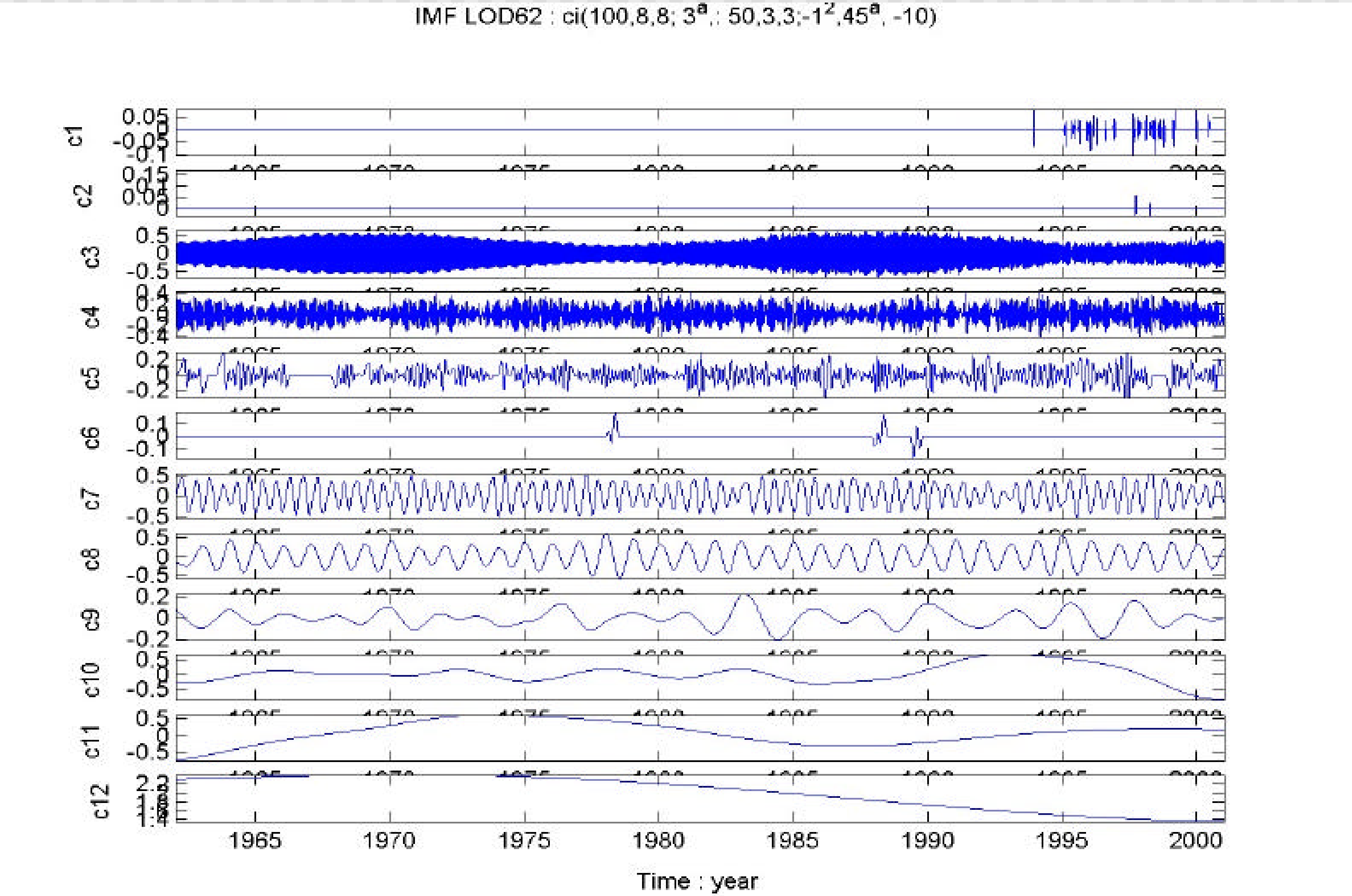


An Example of Sifting

Length Of Day Data



LOD ： IMF

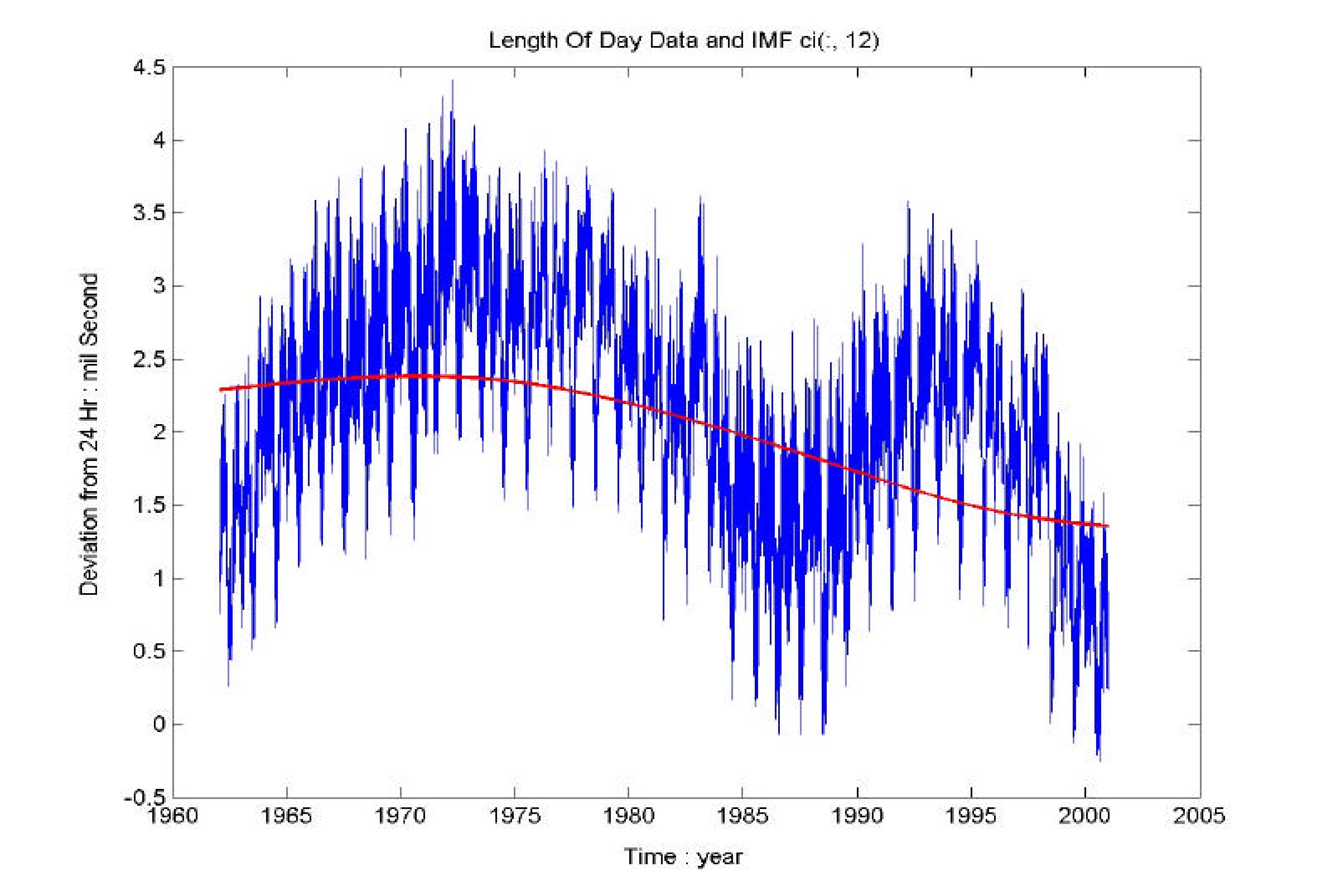

IMF LOD62 : ci(100,8,8; 3ª,: 50,3,3;-1²,45ª, -10)
c1
c2
c3
c4
c5
c6
c7
c8
c9
c10
c11
c12
1965
1970
1975
1980
1985
1990
1995
2000
Time : year

Orthogonality Check

- Pair-wise %
- 0.0003
- 0.0001
- 0.0215
- 0.0117
- 0.0022
- 0.0031
- 0.0026
- 0.0083
- 0.0042
- 0.0369
- 0.0400

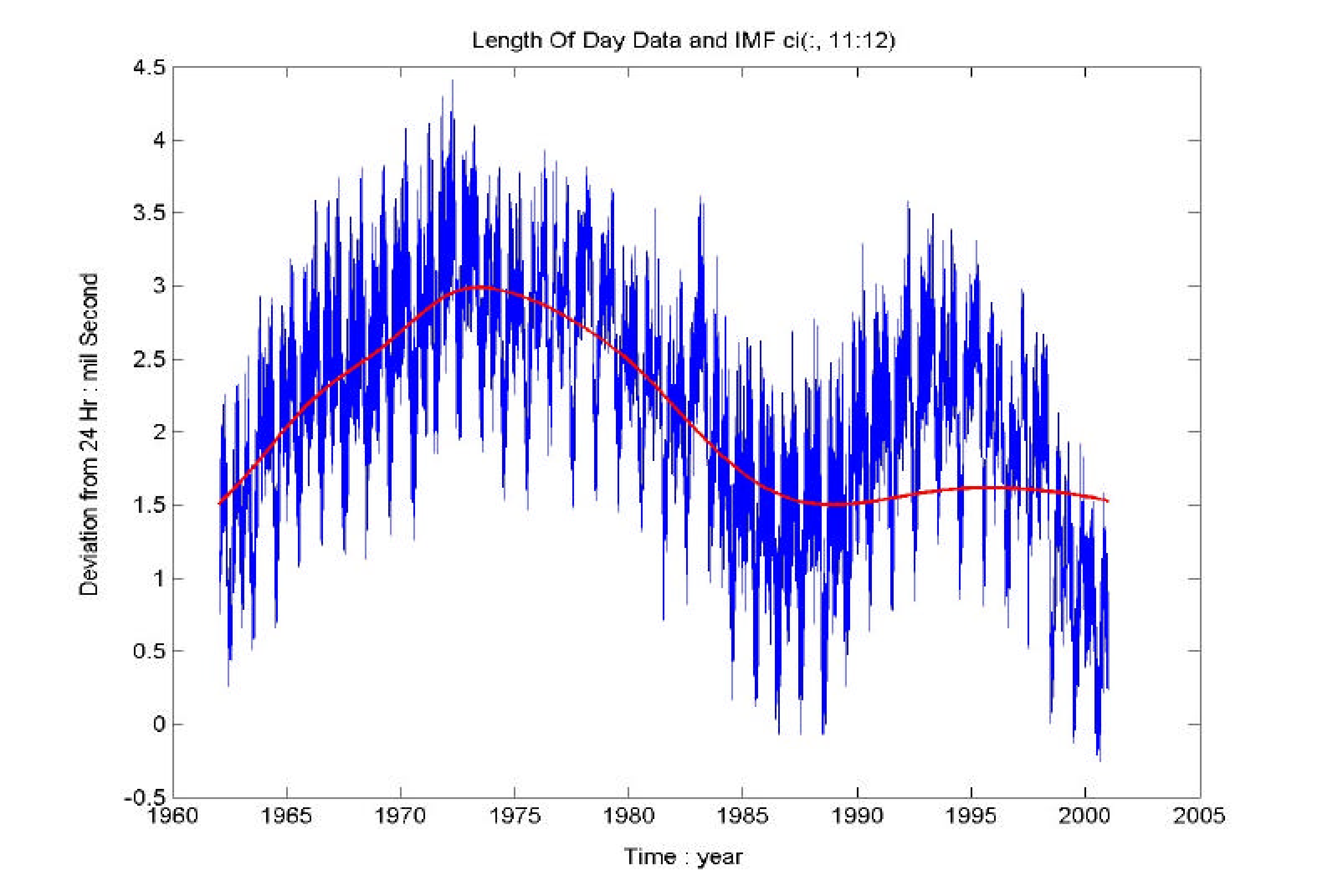
- Overall %
- 0.0452

LOD : Data & c12

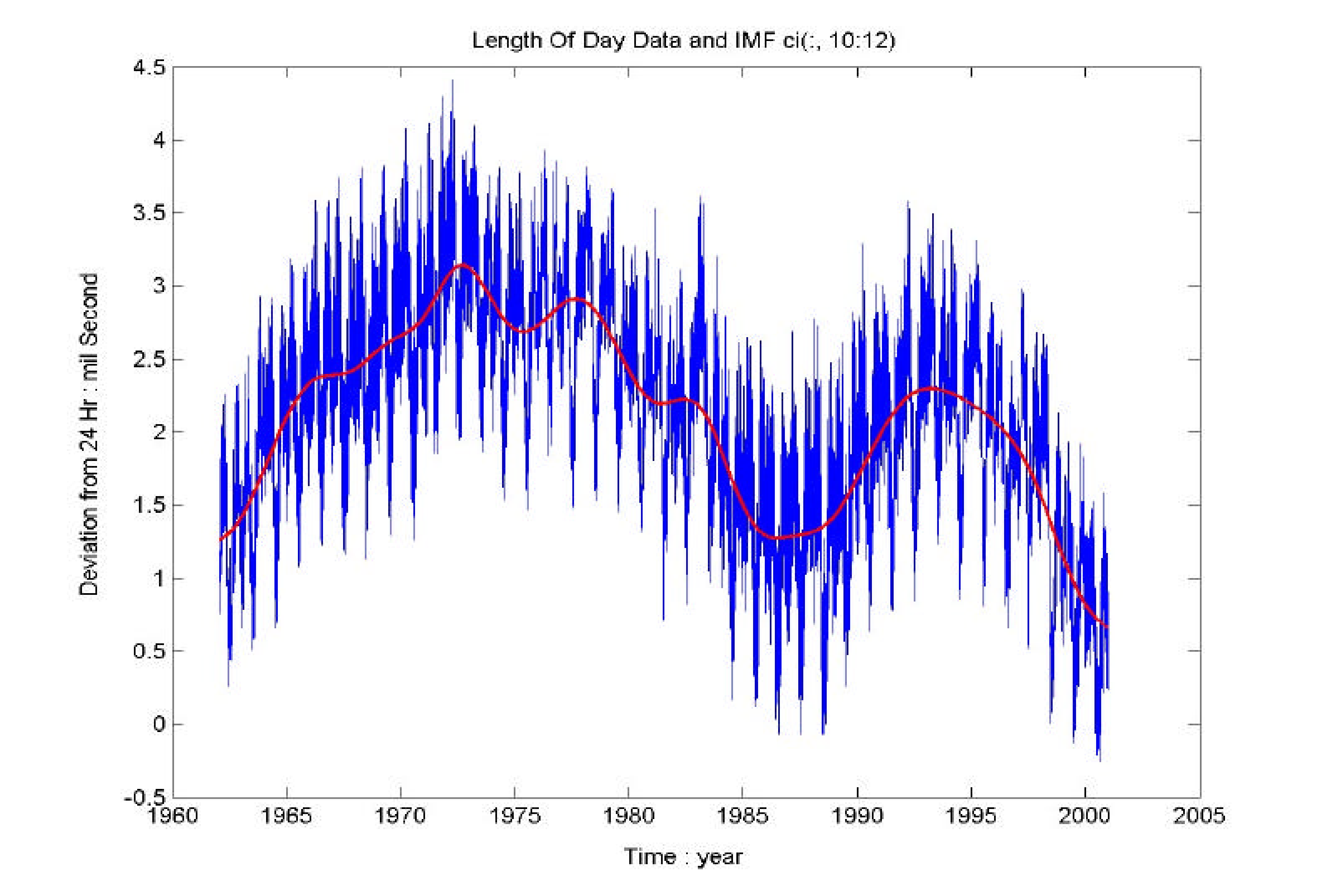


Length Of Day Data and IMF ci(:, 12)
Deviation from 24 Hr : mil Second
Time : year
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

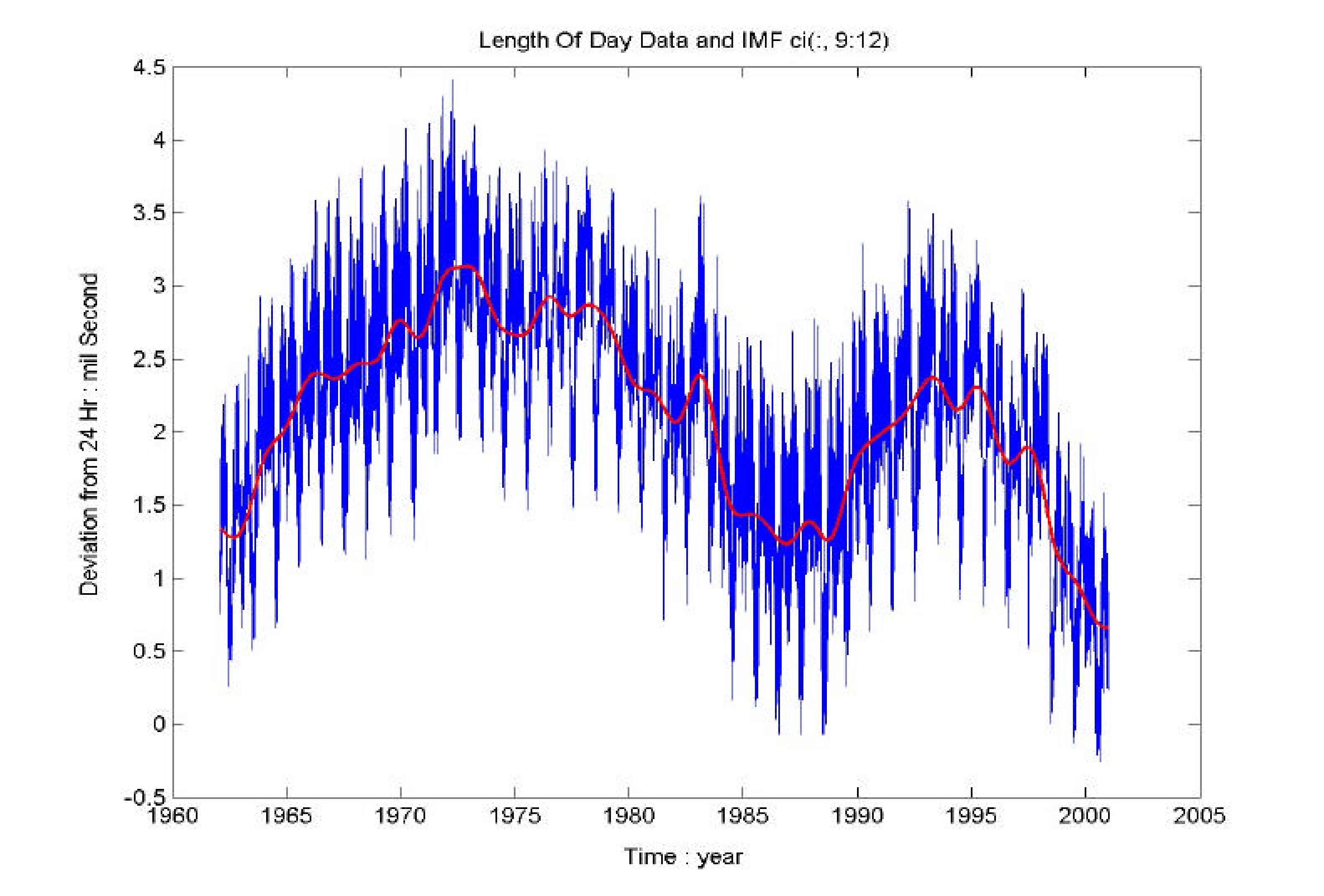
LOD ： Data & Sum c11-12



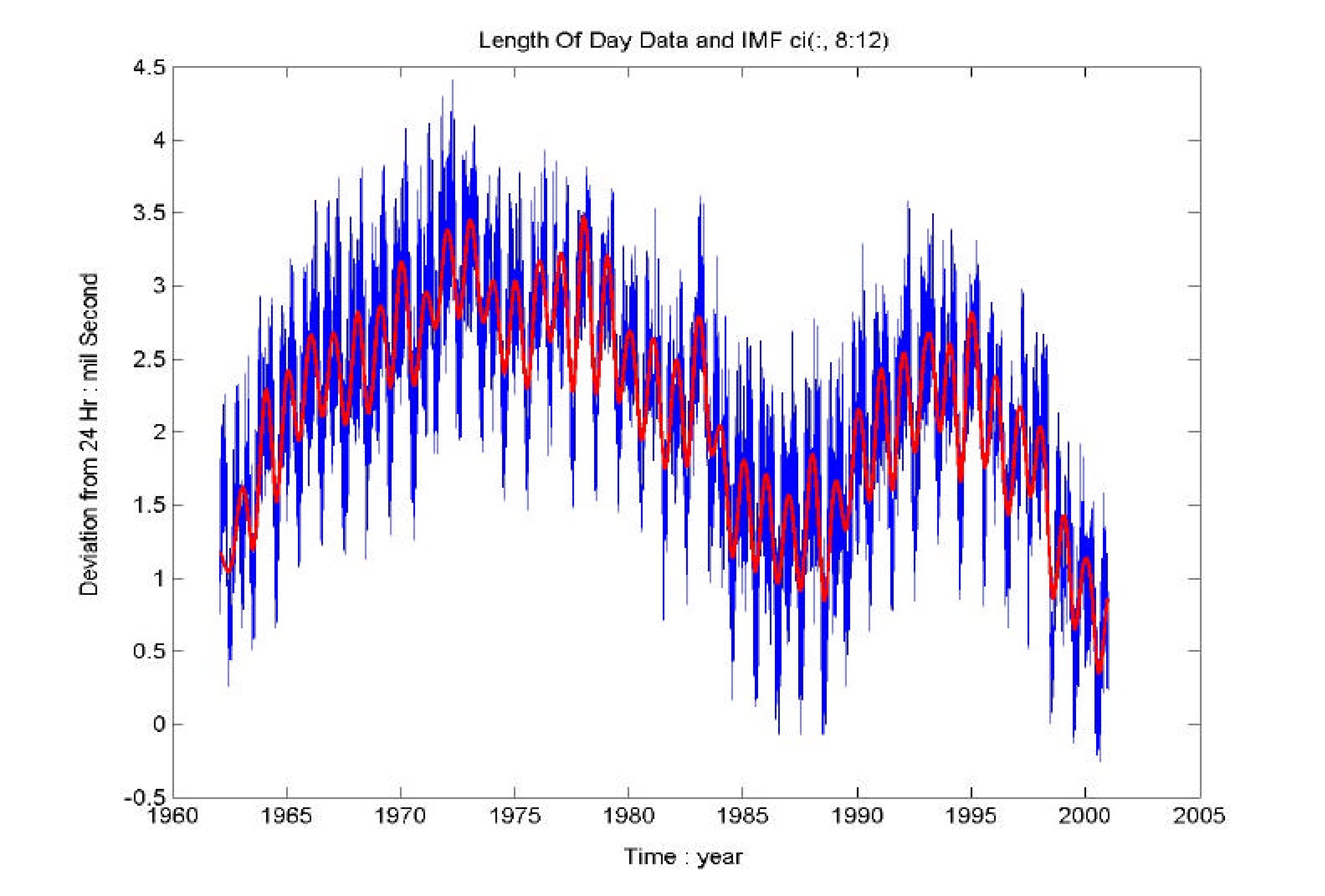
LOD : Data & sum c10-12



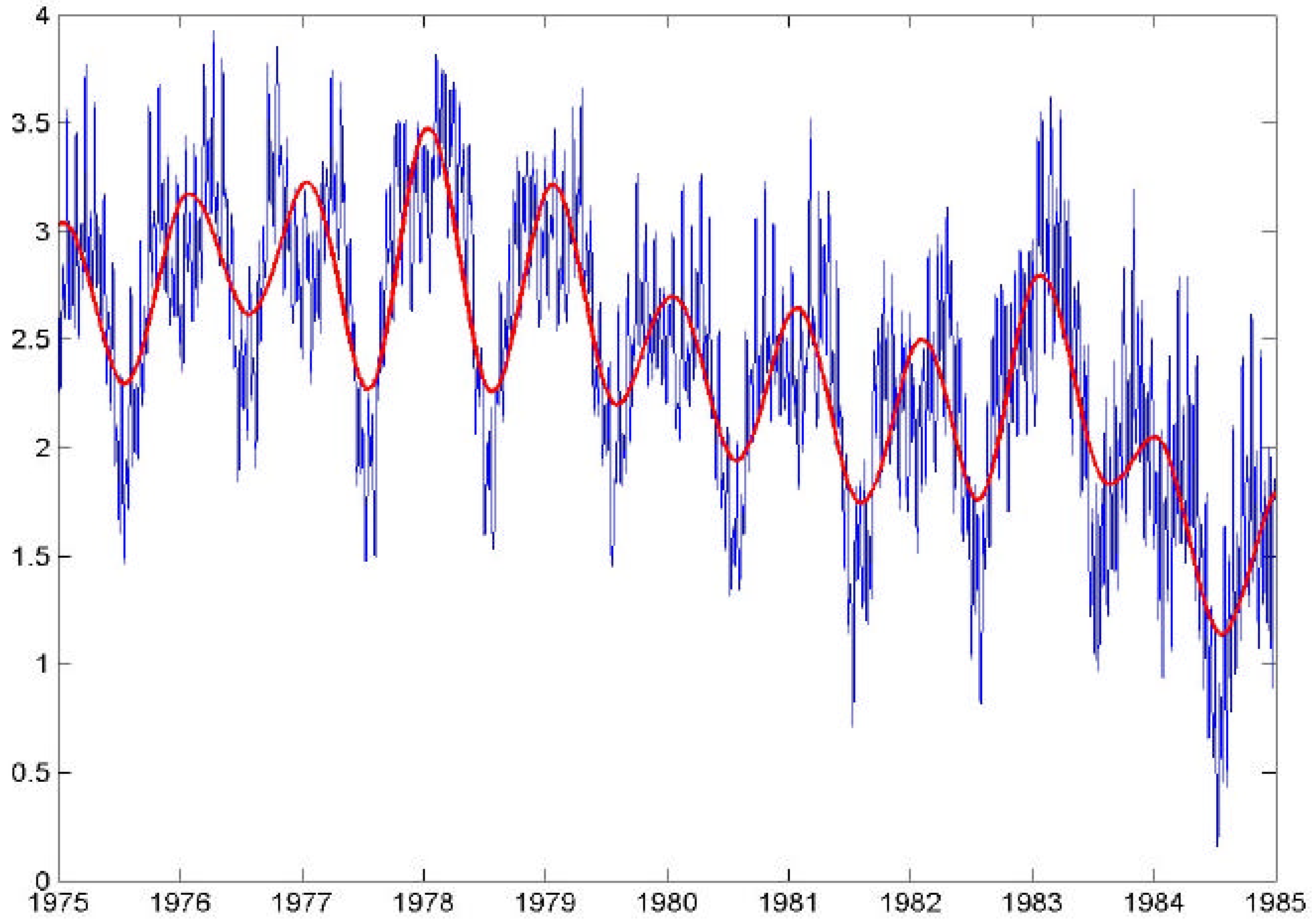
LOD : Data & c9 - 12



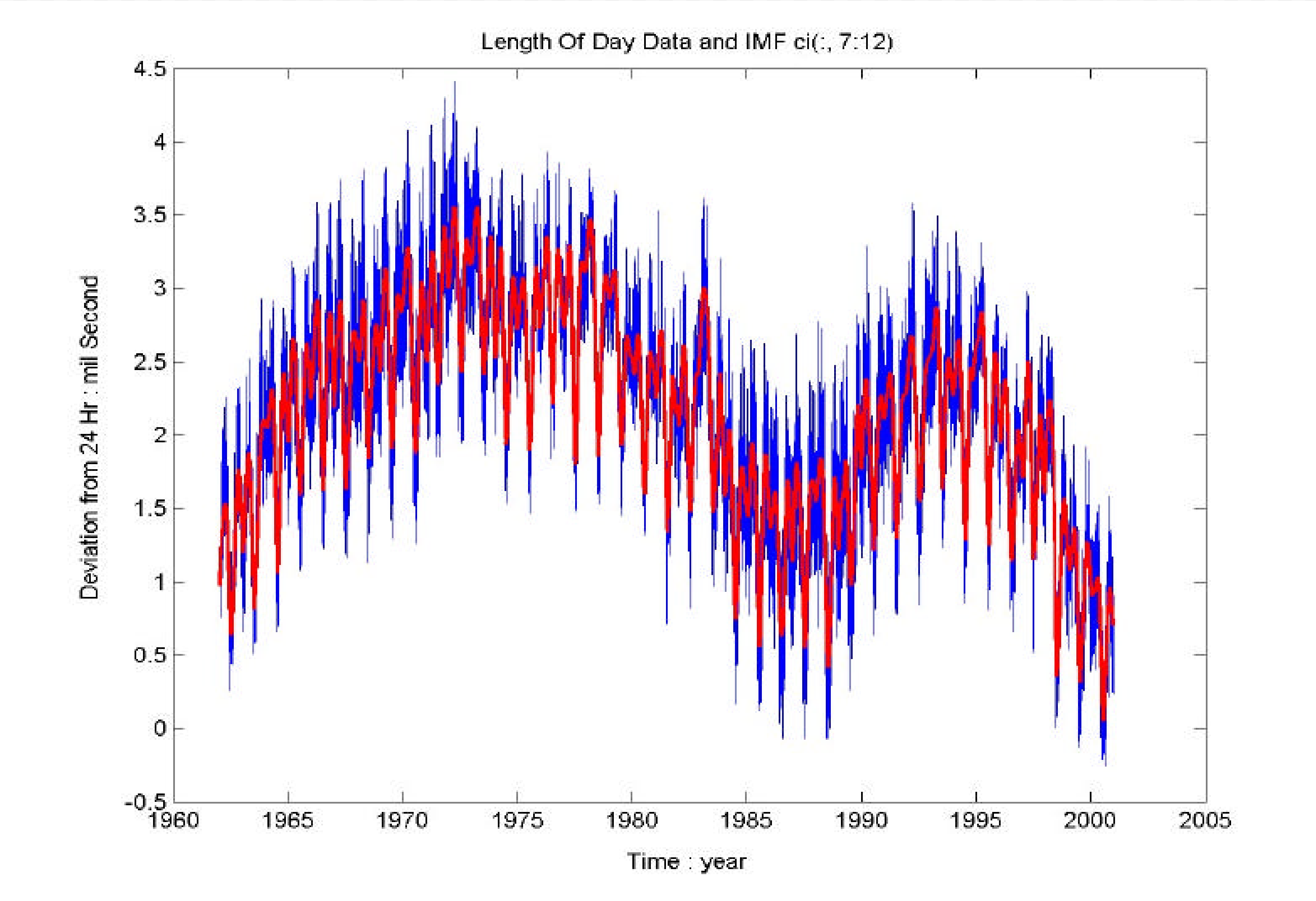
LOD : Data & c8 - 12



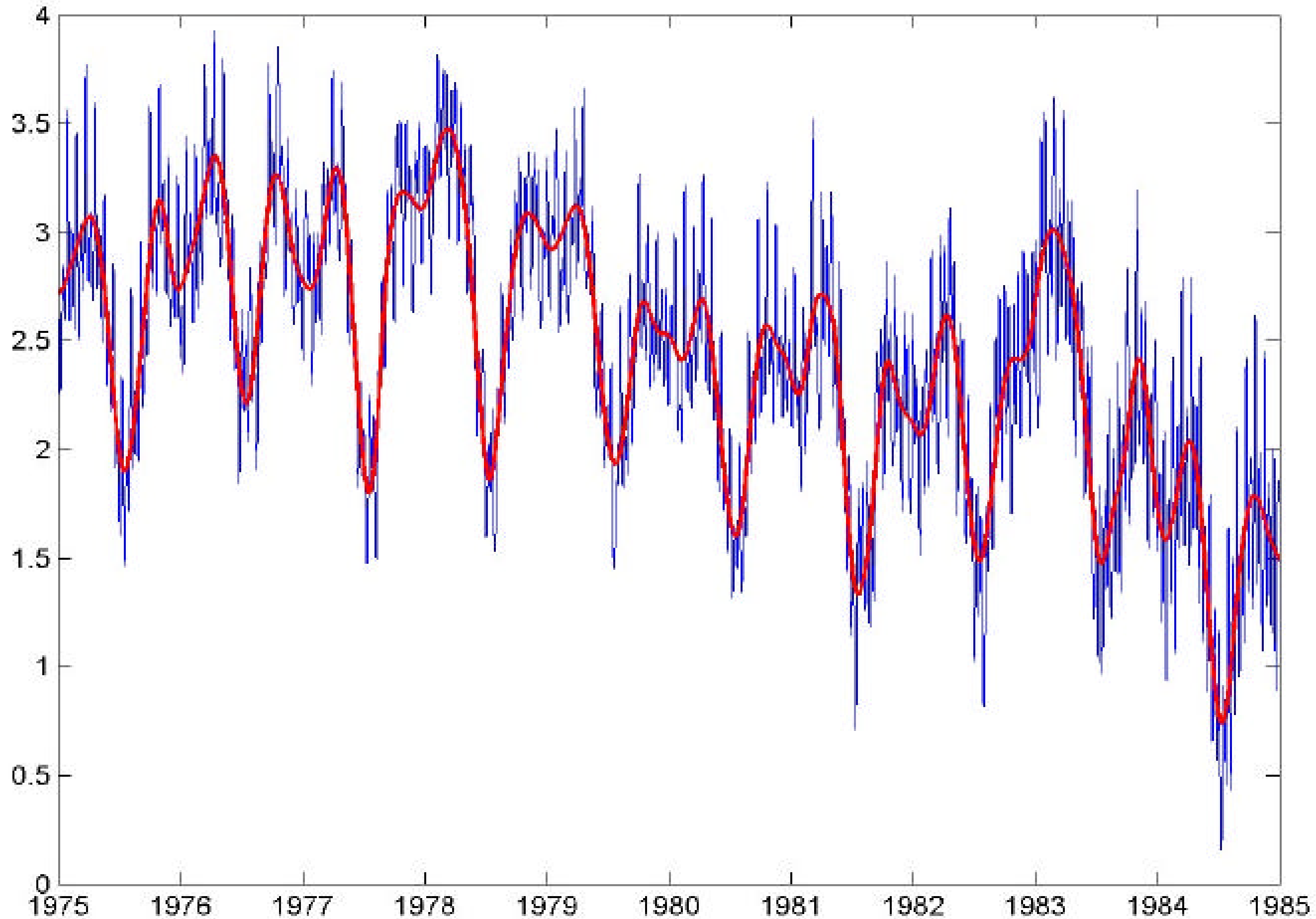
LOD : Detailed Data and Sum c8-c12



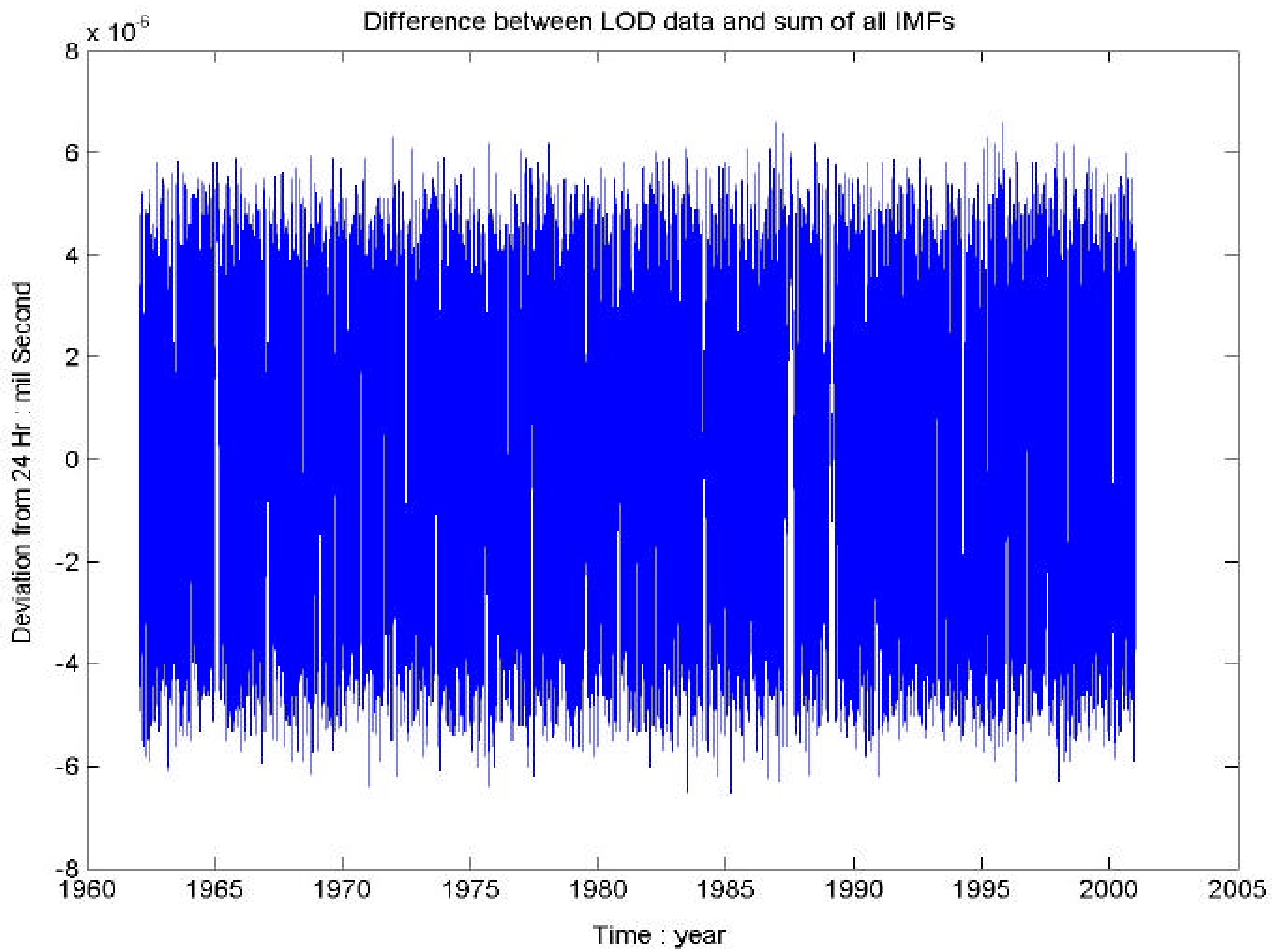
LOD : Data & c7 - 12



LOD : Detail Data and Sum IMF c7-c12

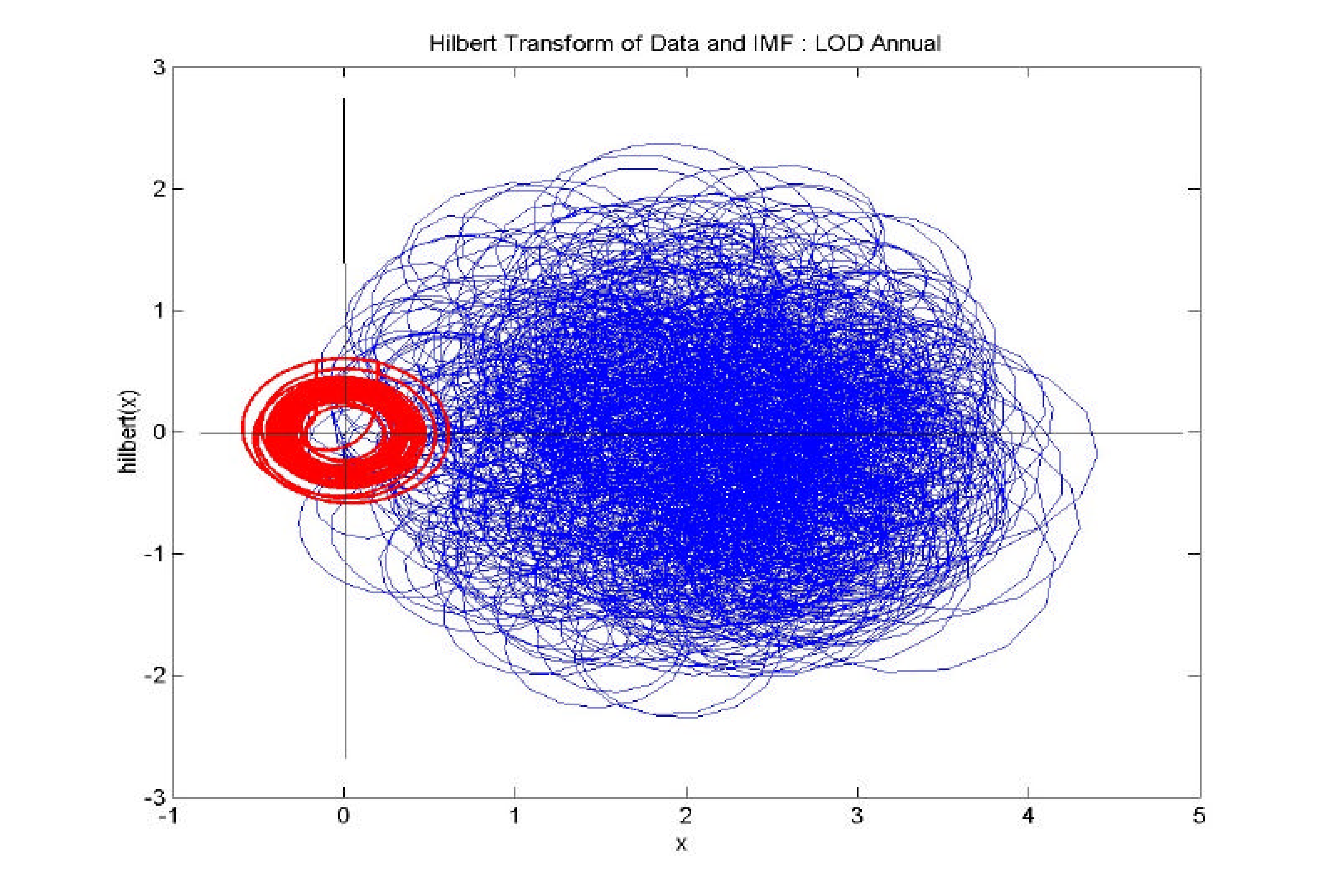


LOD : Difference Data – sum all IMFs

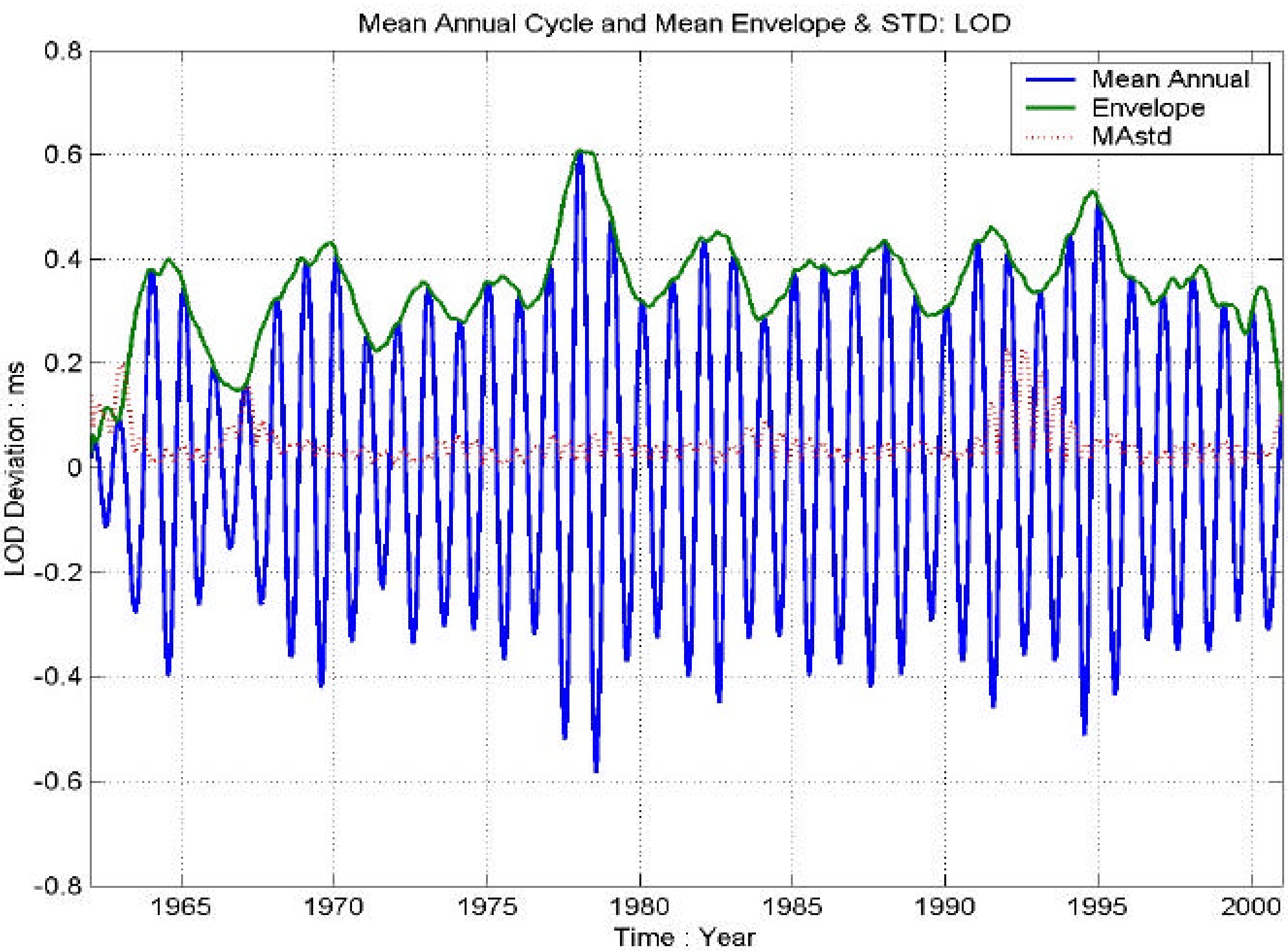


Traditional View

a la Hahn (1995) : Hilbert



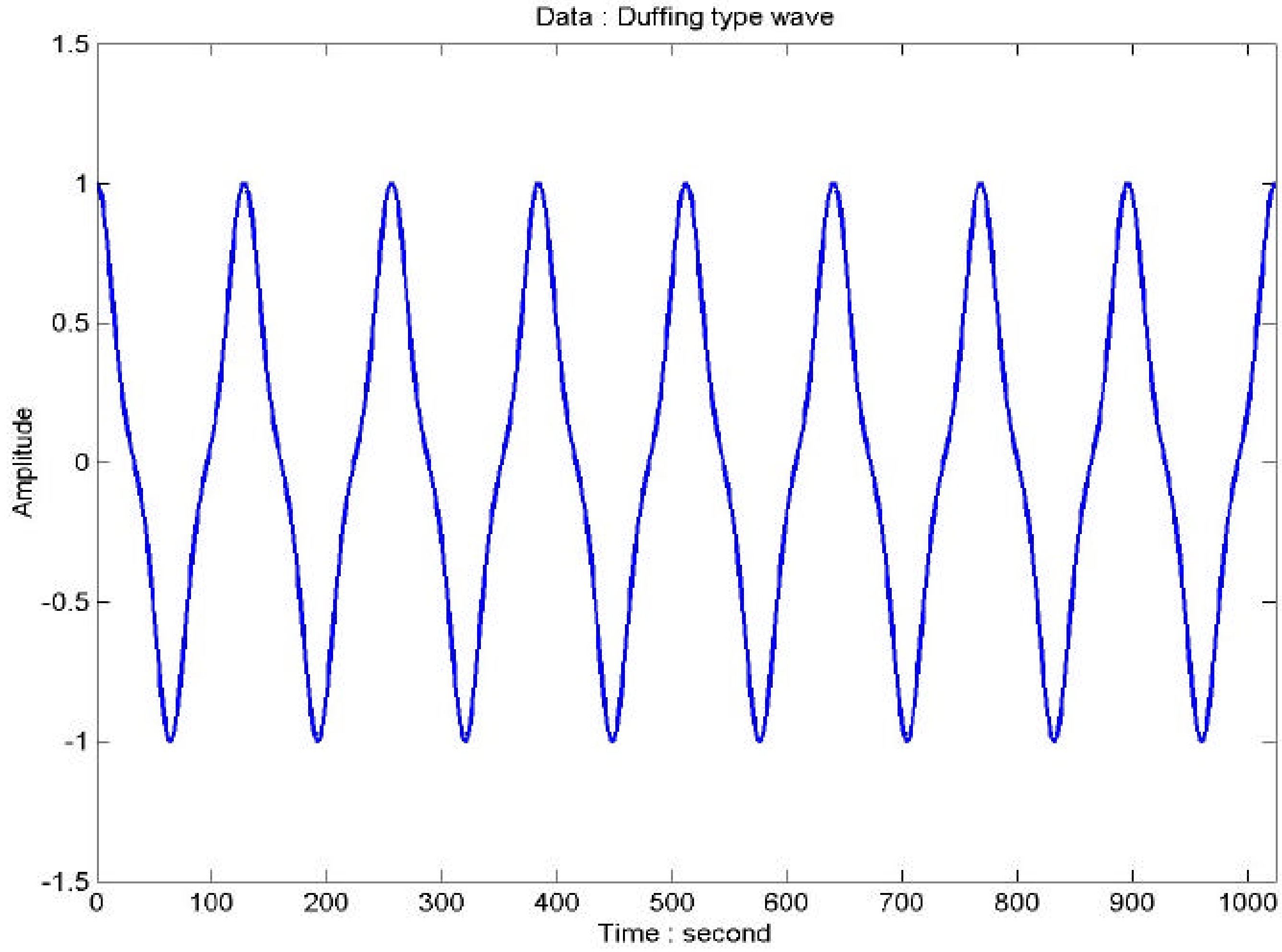
Mean Annual Cycle & Envelope: 9 CEI Cases



Hilbert's View on Nonlinear Data

Duffing Type Wave

Data: $x = \cos(wt + 0.3 \sin 2wt)$



Duffing Type Wave

Perturbation Expansion

For $\varepsilon \ll 1$, *we can have*

$$x(t) = \cos\left(\omega t + \varepsilon \sin 2\omega t\right)$$

$$= \cos\omega t \cos\left(\varepsilon \sin 2\omega t\right) - \sin\omega t \sin\left(\varepsilon \sin 2\omega t\right)$$

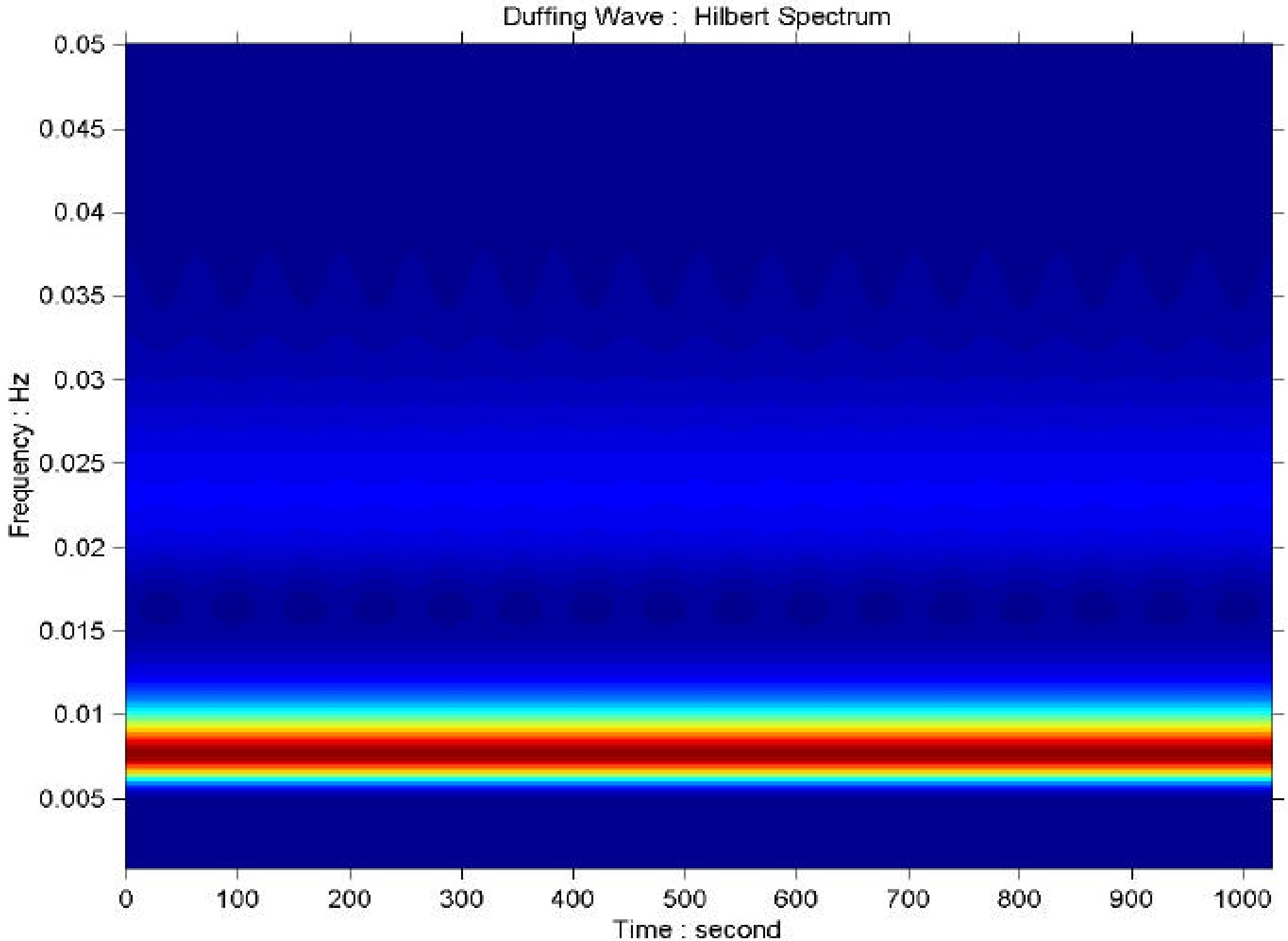
$$= \cos\omega t - \varepsilon \sin\omega t \sin 2\omega t + \ldots.$$

$$= \left(1 - \frac{\varepsilon}{2}\right)\cos\omega t + \frac{\varepsilon}{2}\cos 3\omega t + \ldots.$$

This is very similar to the solution of Duffing equation.

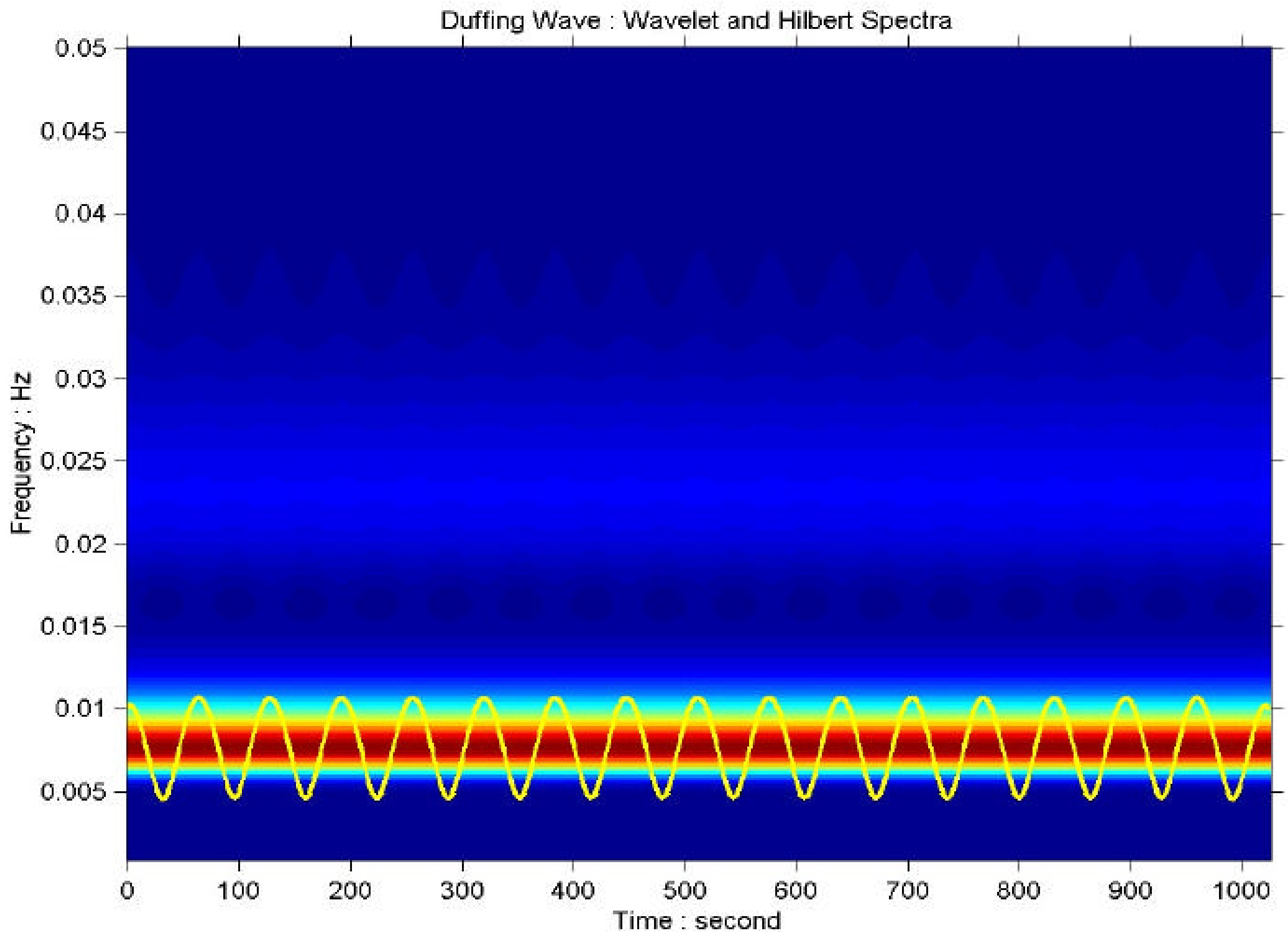
Duffing Type Wave

Wavelet Spectrum

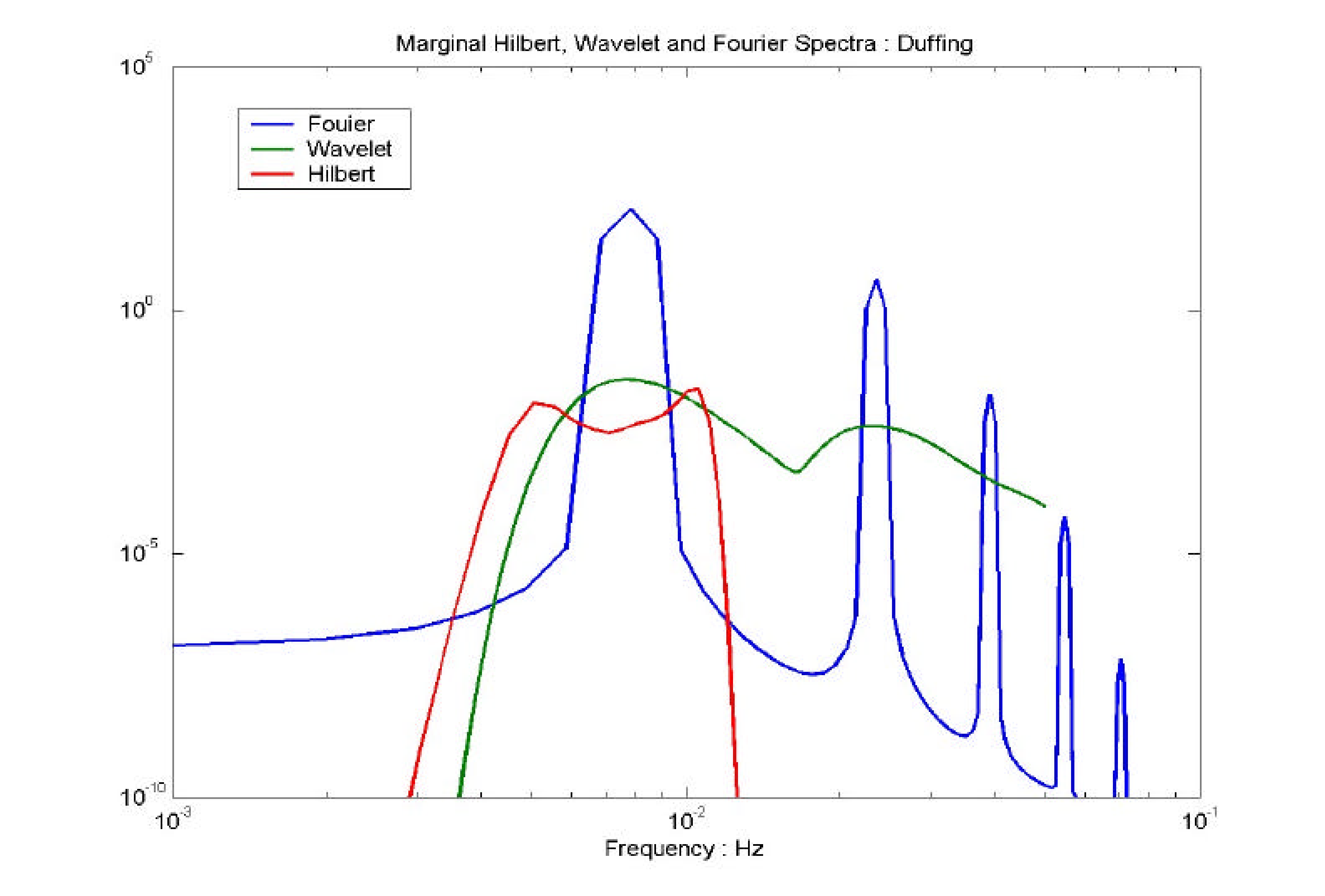


Duffing Type Wave

Hilbert Spectrum



Duffing Type Wave
Marginal Spectra



Duffing Equation

$$\frac{d^2 x}{dt^2} + x + e\,x^3 = g\cos wt\,.$$

*Solved with **ode23tb** for t = 0 to 200 with*

$e = -1$

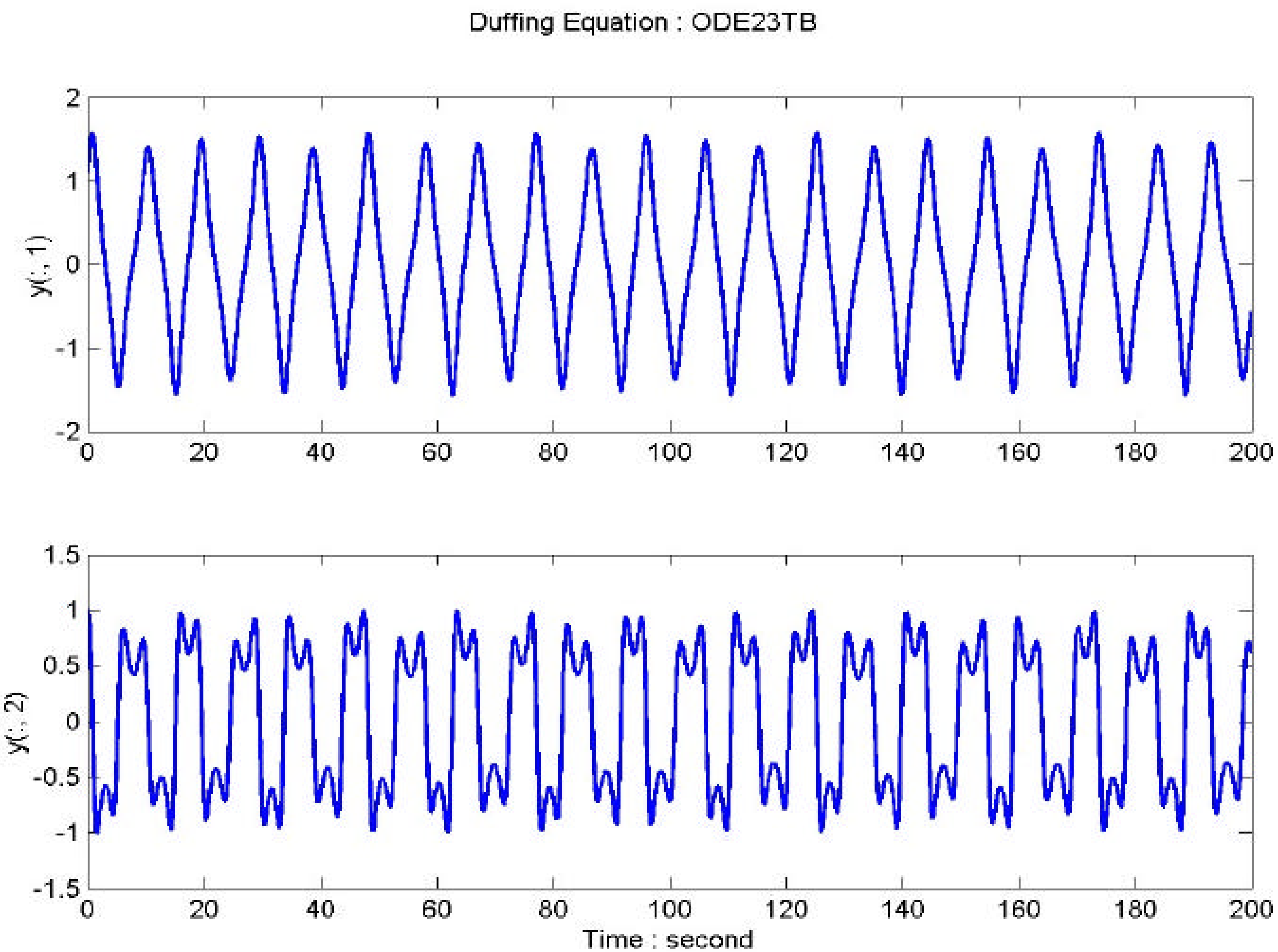
$g = 0.1$

$w = 0.04\ Hz$

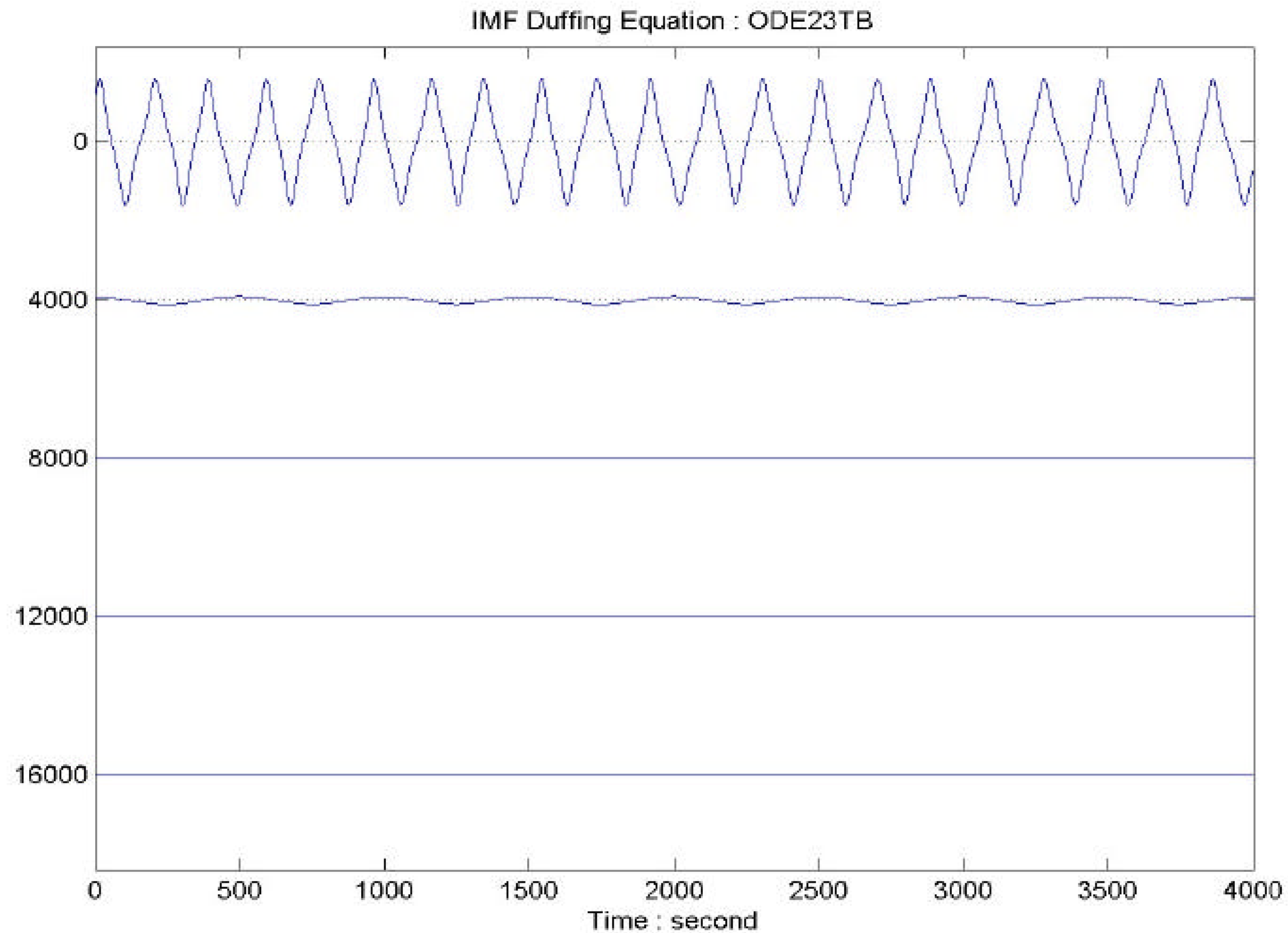
Initial condition :

$[x(o),\ x'(0)] = [1,1]$

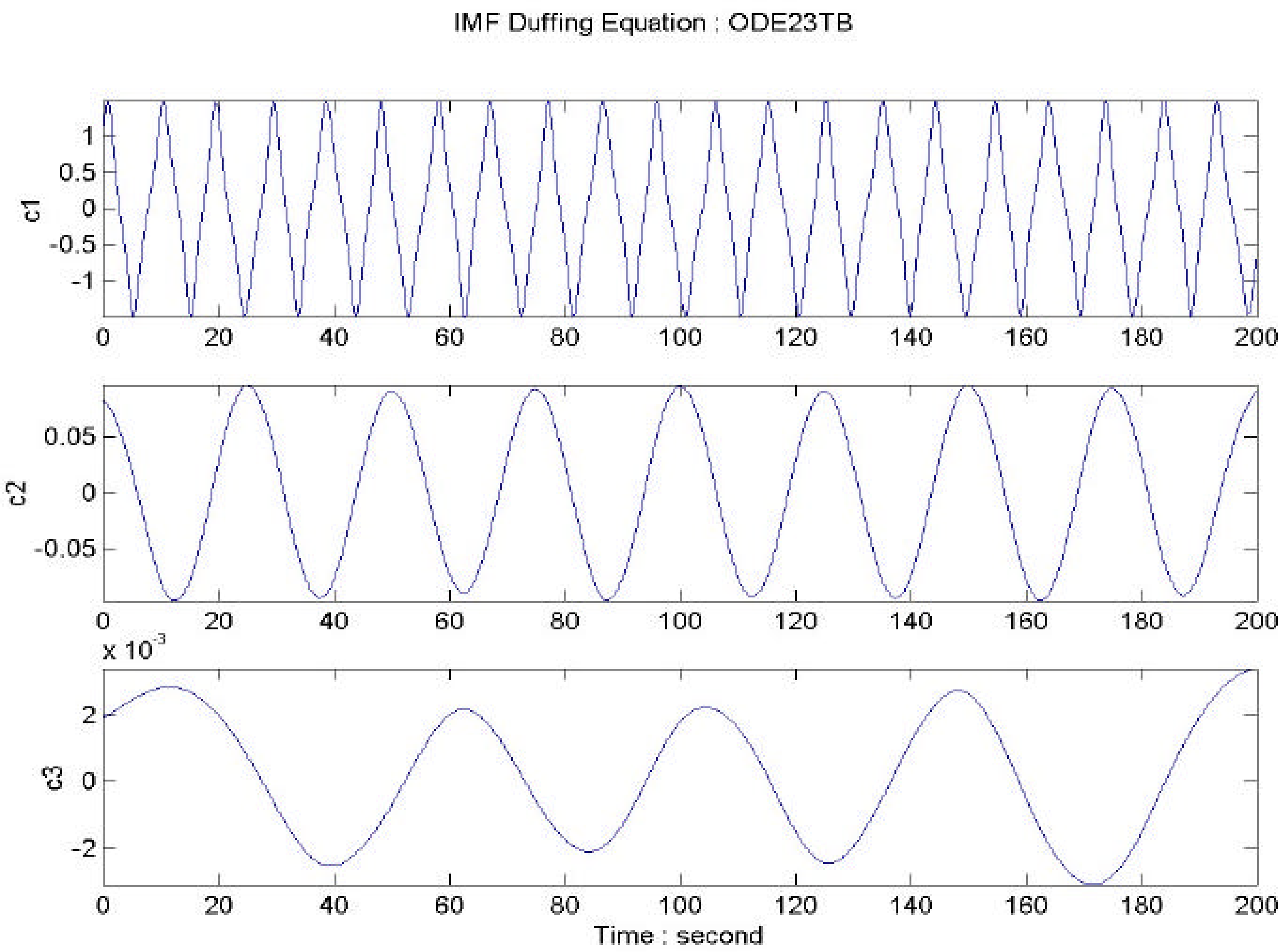
Duffing Equation : Data



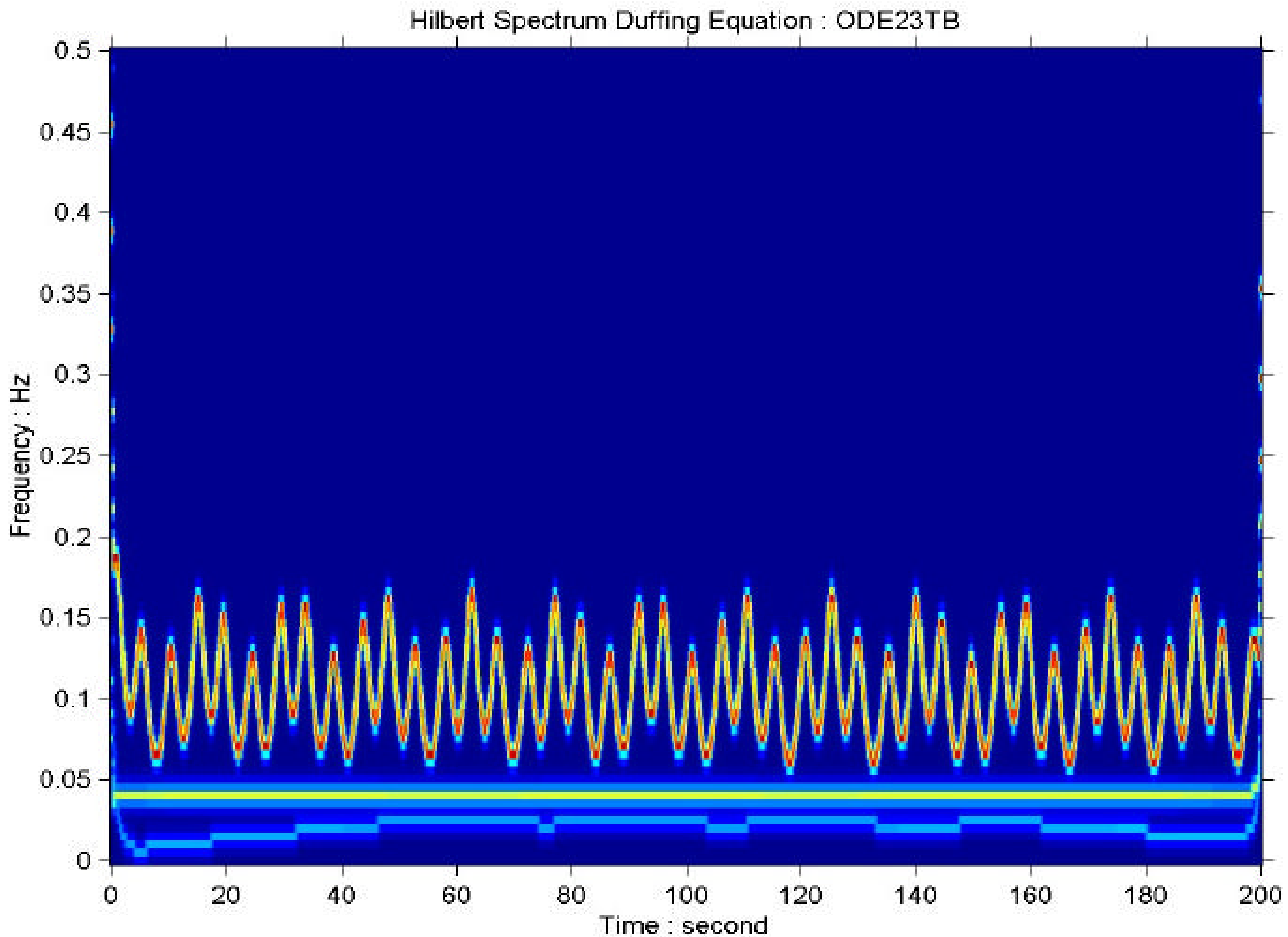
Duffing Equation : IMFs



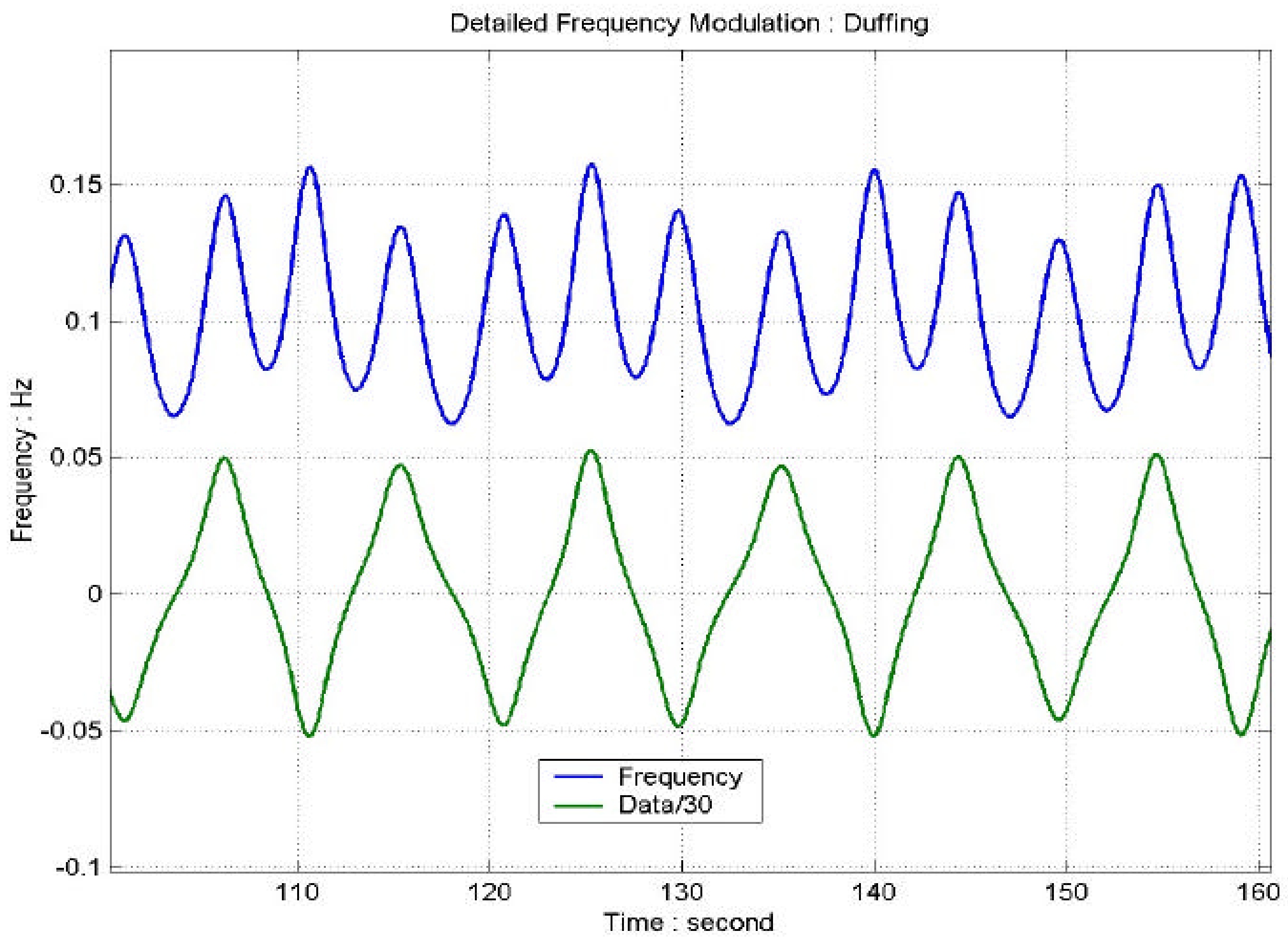
Duffing Equation : IMFs



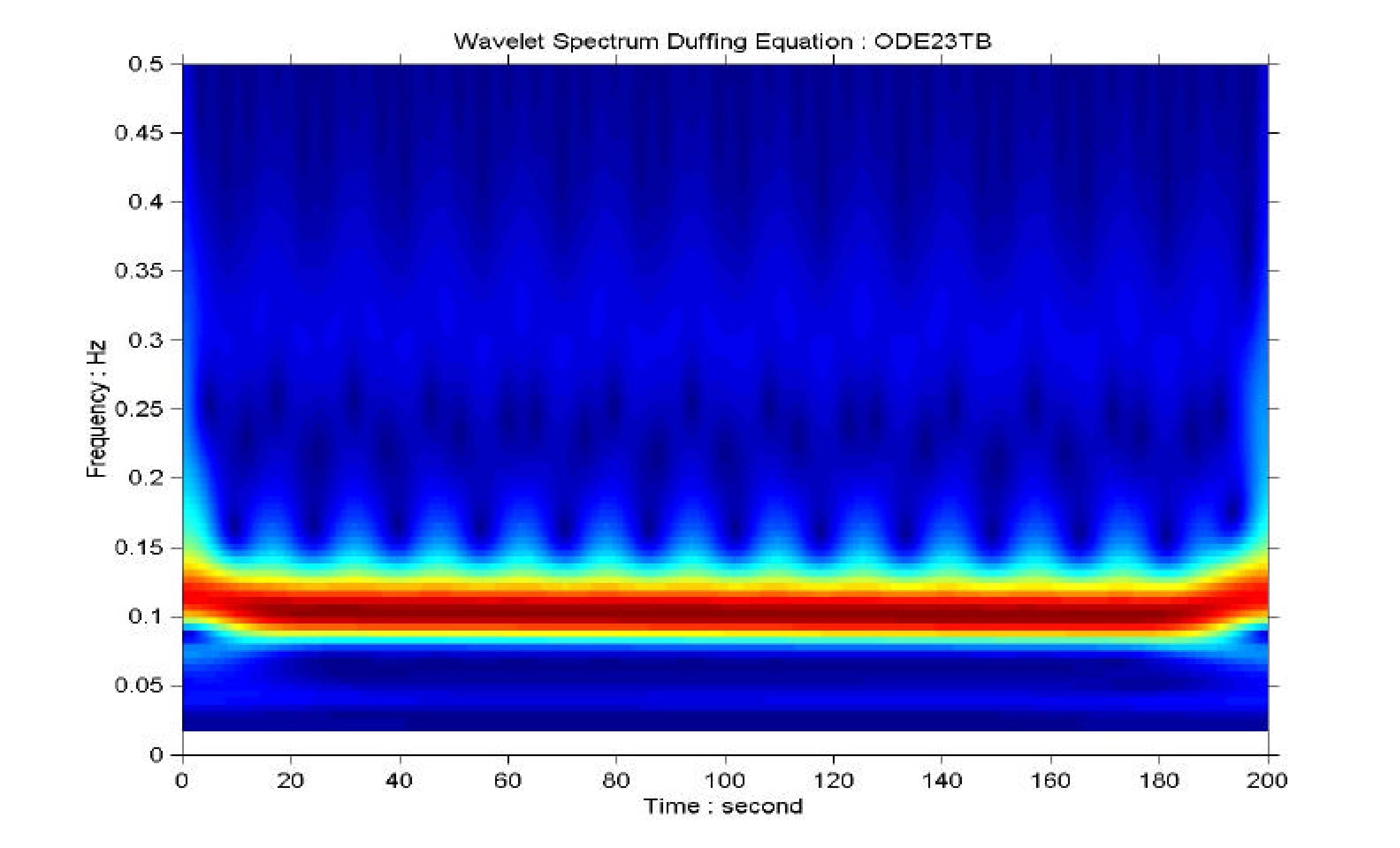
Duffing Equation : Hilbert Spectrum



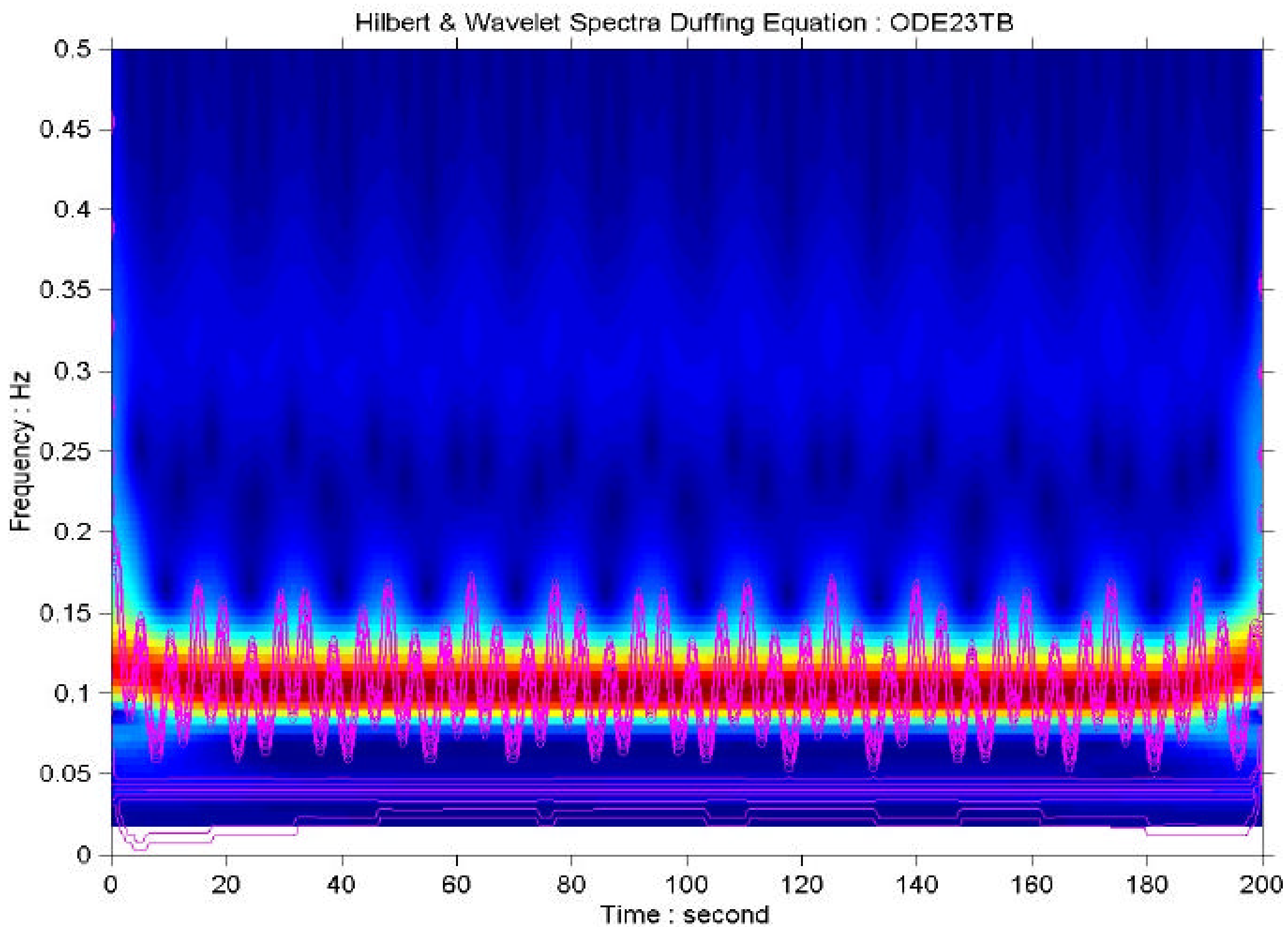
Duffing Equation : Detailed Hilbert Spectrum



Duffing Equation : Wavelet Spectrum



Duffing Equation : Hilbert & Wavelet Spectra



What This Means

- Instantaneous Frequency offers a total different view for nonlinear data: instantaneous frequency with no need for harmonics and unlimited by uncertainty.
- Adaptive basis is indispensable for nonstationary and nonlinear data analysis
- HHT establishes a new paradigm of data analysis

Comparisons

	Fourier	Wavelet	Hilbert
Basis	a priori	a priori	Adaptive
Frequency	Convolution: Global	Convolution: Regional	Differentiation: Local
Presentation	Energy-frequency	Energy-time-frequency	Energy-time-frequency
Nonlinear	no	no	yes
Non-stationary	no	yes	yes
Uncertainty	yes	yes	no
Harmonics	yes	yes	no

Current Applications

- Non-destructive Evaluation for Structural Health Monitoring
  - (DOT, NSWC, and DFRC/NASA, KSC/NASA Shuttle)
- Vibration, speech, and acoustic signal analyses
  - (FBI, MIT, and DARPA)
- Earthquake Engineering
  - (DOT)
- Bio-medical applications
  - (Harvard, UCSD, Johns Hopkins, and Southampton, UK)
- Global Primary Productivity Evolution map from LandSat data
  - (NASA Goddard, NOAA)
- Cosmological Gravity Wave and Planets hunting
  - (NASA Goddard, and Nicholas Copernicus University, Poland)
- Financial market data analysis
  - (NASA and HKUST)